

UNIT III – FREE ELECTRON THEORY & BAND THEORY OF SOLIDS

FREE ELECTRON THEORY

Introduction:

In solids, electrons in outer most orbits of atoms determine its electrical properties. Electron theory is applicable to all solids, both metals and non metals. In addition, it explains the electrical, thermal and magnetic properties of solids. The structure and properties of solids are explained employing their electronic structure by the electron theory of solids. It has been developed in three main stages.

1. Classical free electron theory
2. Quantum Free Electron Theory.
3. Zone Theory.

1. Classical free electron theory: The first theory was developed by Drude and Lorentz in 1900. According to this theory, metal contains free electrons which are responsible for the electrical conductivity and electrons obey the laws of classical mechanics.

2. Quantum Free Electron Theory: In 1928 Sommerfeld developed the quantum free electron theory. According to Sommerfeld, the free electrons move with a constant potential. This theory obeys quantum laws.

3. Zone Theory: Bloch introduced the band theory in 1928. According to this theory, free electrons move in a periodic potential provided by the lattice. This theory is also called “Band Theory of Solids”. It gives complete informational study of electrons.

1) What are the assumptions of classical free electron theory?

Classical free electron theory of metals (Drude – Lorentz theory of metals):

Drude and Lorentz proposed this theory in 1900. According to this theory, the metals containing the free electrons obey the laws of classical mechanics.

Assumptions (or) Salient features in classical free electron theory

1. In metals there are a large number of free electrons moving freely in all possible directions.
2. These free electrons behave like gas molecules in a container obeying the laws of kinetic theory of gases.
3. In the absence of field the energy associated with each electron at a temperature T is given by $\frac{3}{2}kT$. It is related to kinetic energy as $\frac{3}{2}kT = \frac{1}{2}mv_{th}^2$. Where v_{th} is the thermal velocity and k is Boltzmann constant.
4. In metals, the positive ion cores are at fixed positions and the free electrons move randomly and collide either with positive ion cores or with other free electrons or with boundaries. Hence these collisions are elastic. Therefore the electric conduction is due to free electrons only.
5. Electron velocities in a metal obey Maxwell-Boltzmann distribution of velocities.
6. The free electrons move in a constant potential field. Hence the potential energy of the

electrons is constant.

7. When an electric field is applied to a metal, free electrons are accelerated in the direction opposite to the direction of applied electric field with a velocity called drift velocity represented as v_d .

2. What are the merits and demerits of classical free electron theory?

Success or Advantages or Merits classical free electron theory

- 1) It verifies ohm's law.
- 2) It explains electrical and thermal conductivities of metals.
- 3) It derives Widemann-Franz law.
- 4) It explains optical properties of metals.

Limitations or drawbacks or Demerits classical free electron theory

- 1) It fails to explain the electrical conductivity of semiconductors and insulators.
- 2) It fails to explain the temperature variation of electrical conductivity at low temperature.
- 3) It fails to explain the concept of specific heat of metals.
- 4) It fails to explain the mean free path of the electrons.
- 5) The phenomenon like photo electric effect, Compton effect and black body radiation could not be explained by classical free electron theory.
- 6) It fails to explain temperature dependence of paramagnetic susceptibility and ferromagnetism.

3) What are the assumptions of quantum free electron theory?

Quantum Free Electron Theory:

Quantum free electron theory was proposed by Sommerfeld in 1928. It overcomes many of the drawbacks of classical theory. Sommerfeld explained them by choosing Fermi-Dirac statistics instead of Maxwell-Boltzmann statistics. He developed this theory by applying the principles of quantum mechanics.

Assumptions of Quantum Free Electron Theory

- 1) Valence electrons move freely in a constant potential within the boundaries of metal and is prevented from escaping the metal at the boundaries (high potential). Hence the electron is trapped in a potential well.
- 2) The distribution of electrons in various allowed energy levels occurs as per Pauli Exclusion Principle.
- 3) The attraction between the free electrons and lattice ions and the repulsion between electrons themselves are ignored.
- 4) The distribution of energy among the free electrons is according to Fermi-Dirac statistics.
- 5) The energy values of free electrons are quantized.
- 6) To find the possible energy values of electron Schrodinger time independent wave equation is applied. The problem is similar to that of particle present in a potential box.

Energy of electron is $E_n = \frac{n^2 h^2}{8mL^2}$, Where $n=1, 2, 3, \dots$

4. What are the merits and demerits of quantum free electron theory?

Merits of quantum free electron theory

1. It successfully explains the electrical and thermal conductivity of metals.
2. It can explain the Thermionic phenomenon.
3. It explains temperature dependence of conductivity of metals.
4. It can explain the specific heat of metals.
5. It explains magnetic susceptibility of metals.
6. It can explain photo electric effect, Compton Effect and block body radiation etc.
7. It gives the correct mathematical expression for the thermal conductivity of metals.

Demerits of quantum free electron theory

1. It is unable to explain the metallic properties exhibited by only certain crystals.
2. It is unable to explain why the atomic arrays in metallic crystals should prefer certain structures only.
3. This theory fails to distinguish between metal, semiconductor and Insulator.
4. It also fails to explain the positive value of Hall Co-efficient.
5. According to this theory, only two electrons are present in the Fermi level and they are responsible for conduction which is not true.

5. Derive the expression for electrical conductivity on the basis of quantum free electron theory.

Expression for electrical conductivity:

When an electric field E is applied, the force on the electron is $-eE$. We know that force is also rate of change of momentum dp/dt .

$$\text{Therefore } -eE = \frac{dp}{dt} \text{-----(1)}$$

Since $p = h/\lambda = (h/2\pi)(2\pi/\lambda) = \hbar k$. Where $k = 2\pi/\lambda$.

$$\Rightarrow \frac{dp}{dt} = \hbar \frac{dk}{dt} \text{-----(2)}$$

\Rightarrow Substituting (2) in (1), we have

$$\Rightarrow -eE = \hbar \frac{dk}{dt}$$

$$\Rightarrow dk = -eE \frac{dt}{\hbar}$$

\Rightarrow Origin of k space moves through a distance dk in time dt . The displacement in the average collision time τ is $\Delta k = -\frac{eE\tau}{\hbar}$.

Since $p = mv = \hbar k \rightarrow m\Delta v = \hbar\Delta k$

$$\Rightarrow \Delta k = \frac{m\Delta v}{\hbar}$$

$$\Rightarrow \frac{m\Delta v}{\hbar} = -\frac{eE\tau}{\hbar}$$

$$\Rightarrow \Delta v = -\frac{eE\tau}{m}$$

Expression for current density is $J = -ne\Delta v$

Substituting Δv in the above equation, we have

$$J = -ne\left[-\frac{eE\tau}{m}\right]$$

$$\Rightarrow J = \frac{ne^2E\tau}{m}$$

Also we have $J = \sigma E$. Comparing these two equations we have $\sigma E = \frac{ne^2E\tau}{m}$

Therefore the expression for electrical conductivity is given by

$$\sigma = \frac{ne^2\tau}{m}$$

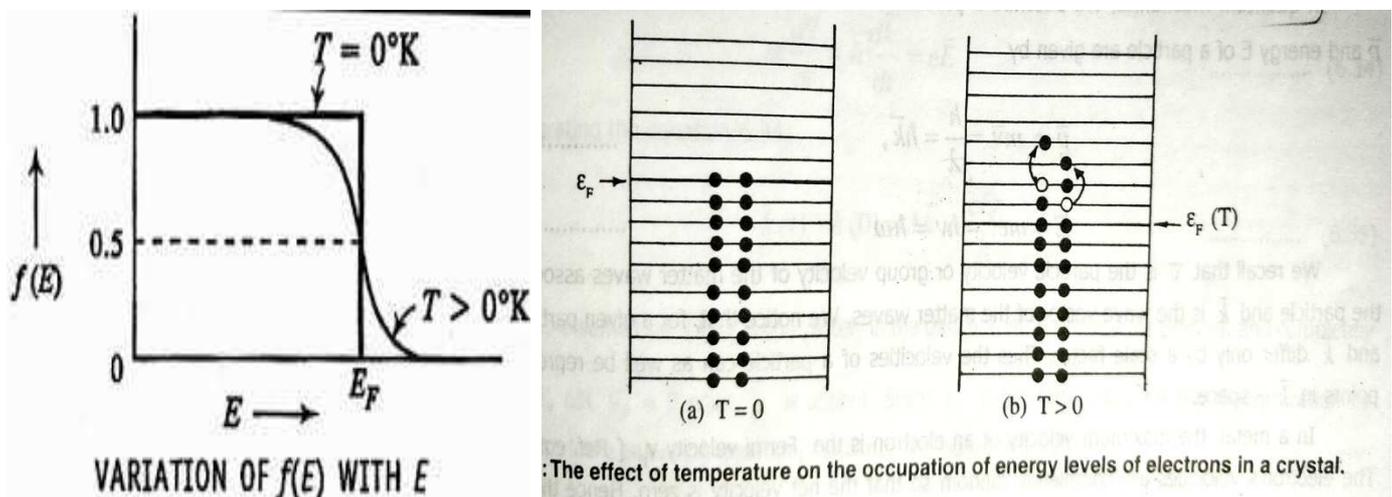
6. Explain Fermi-Dirac distribution function.

Fermi Dirac Distribution:

A metal piece contains very large number of electrons. Each electron possesses quantized energy states and obeys Pauli's exclusion principle. Hence they satisfy Fermi-Dirac statistics. The probability $F(E)$ of an electron occupying energy level E_i is given by

$$F(E_i) = \frac{1}{\exp\left(\frac{E_i - E_f}{kT}\right) + 1}$$

Where $F(E_i)$ is called Fermi function which is defined as the probability of electron occupation in the given energy state (E_i) at thermal equilibrium. E_f is Fermi energy, E_i is energy of i^{th} state and k is Boltzmann constant.



The plot of $F(E)$ Vs E is as shown in fig.

Conclusions: At $T=0K$, the Fermi Dirac distribution of electrons can be understood mathematically from the following two cases.

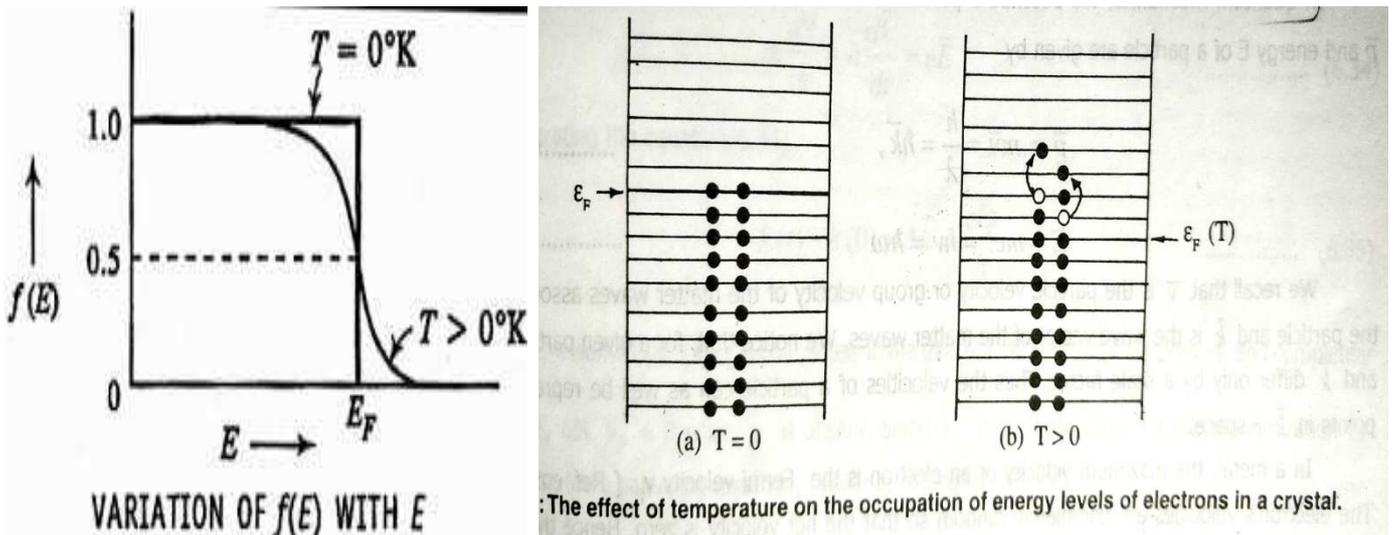
Case -1: If $E > E_F$, $e^{\frac{(E_i - E_F)}{kT}} = \infty$

Therefore, $F(E_i) = 1/\infty$ then $F(E_i) = 0$. It indicates that energy levels above Fermi level are empty.

Case -2: If $E < E_F$, $e^{\frac{(E_i - E_F)}{kT}} = 0$ then $F(E_i) = 1$. It indicates that energy levels below Fermi level are full of electrons. The Fermi level is a boundary energy level which separates the filled energy state and empty states at $0k$. The energy of the highest filled state at $0K$ is called Fermi Energy E_F and the energy level is known as Fermi Level.

Case-3: At $T > 0K$, if $E_i = E_F$ then $F(E_i) = 1/2$. i.e 50%. Therefore Fermi level is the energy level for which the probability of filled states is 50% at any temperature. If T_F is the Fermi temperature and V_F is Fermi velocity corresponding to Fermi energy E_F then $E_F = kT_F \Rightarrow T_F = E_F / k$ and $V_F = \sqrt{(2E_F/m)}$.

7. Explain the effect of temperature on Fermi-Dirac distribution.

Effect of temperature on Fermi Dirac distribution:

1. At **$T=0\text{K}$** , the Fermi Dirac distribution of electrons can be understood mathematically from the following two cases.

Case -1: If $E > E_F$, $e^{\frac{(E_i - E_F)}{kT}} = \infty$

Therefore, $F(E_i) = 1/\infty$ then $F(E_i) = 0$. It indicates that energy levels above Fermi level are empty.

Case -2: If $E < E_F$, $e^{\frac{(E_i - E_F)}{kT}} = 0$ then $F(E_i) = 1$. It indicates that energy levels below Fermi levels are full of electrons.

2. When **$T > 0$** , $F(E_F) < 1$ for $E_i < E_F$

$$F(E_i) > 0 \text{ for } E_i > E_F$$

$$F(E_i) = \frac{1}{2} \text{ for } E_i = E_F$$

As temperature increases more and more electrons jump to the levels above E_F leaving vacancies as shown in the fig.

3. The electrons in the levels above E_F and vacancies in the levels below E_F are responsible for conduction in semiconductors.

4. If the temperature is raised further, the resistance of the metals increases due to decrease of mobility.

5. At **$T > 0\text{K}$** , the Fermi energy level decreases.

6. According to classical theory, all the free electrons below E_F gain energy when temperature is increased. This deviates theoretical and experimental values.

7. According to quantum theory only few electrons below E_F gain energy from external source. Thus theoretical values are closer to the experimental values.

8. Derive an expression for density of energy states.

Density of States :

The number of electronic states per unit energy range is called density of states of $g(E)$. Let us consider a spherical system of radius n and it represents a vector to a point n_x, n_y and n_z ($n^2 = n_x^2 + n_y^2 + n_z^2$) in 3-dimensional space.

Consider a sphere of radius n and another sphere of radius $(n + dn)$ in which energy values are E and $E + dE$ respectively. as shown in fig.

Number of energy states available in sphere of radius n is $\frac{1}{8} \left(\frac{4}{3} \pi n^3 \right)$ -----(1)

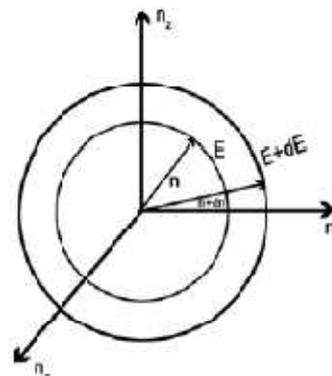
Number of energy states in sphere of radius $(n + dn)$ is $\frac{1}{8} \left(\frac{4}{3} \pi (n + dn)^3 \right)$ -----(2)

E and dE are energy values of above two regions respectively from eq (1) & (2) we can write number of energy states available in dn or dE region as

$$\begin{aligned} g^1(E)dE &= \frac{1}{8} \left[\frac{4}{3} \pi (n + dn)^3 \right] - \frac{1}{8} \left[\frac{4}{3} \pi n^3 \right] \\ &= \frac{\pi}{6} [(n + dn)^3 - n^3] \\ &= \frac{\pi}{6} [n^3 + dn^3 + 3n^2 dn + 3ndn^2 - n^3] \end{aligned}$$

Neglecting higher order terms we get

$$\begin{aligned} g^1(E)dE &= \frac{\pi}{6} (3n^2 dn) \\ &= \frac{\pi}{2} (n^2 dn) \text{-----(3)} \end{aligned}$$



The expression for nth energy level can be written as

$$E = \frac{n^2 h^2}{8ma^2}$$

$$n^2 = \left[\frac{8ma^2}{h^2} \right] E \text{ ----- (4)}$$

$$n = \left[\frac{8ma^2}{h^2} E \right]^{\frac{1}{2}} \text{ ----- (5)}$$

Differentiating eq (4), taking n and E are variables we get

$$2ndm = \frac{8ma^2}{h^2} dE$$

$$ndn = \frac{1}{2} \left(\frac{8ma^2}{h^2} \right) dE \text{ ----- (6)}$$

Substituting eq (5) and(6) in eq (3) we get,

$$g^1(E)dE = \frac{\pi}{2} \left(\frac{8ma^2}{h^2} \right)^{\frac{1}{2}} E^{\frac{1}{2}} \frac{1}{2} \left(\frac{8ma^2}{h^2} \right) dE$$

$$= \frac{\pi}{4} \left(\frac{8ma^2}{h^2} \right)^{\frac{3}{2}} E^{\frac{1}{2}} dE$$

According to Pauli exclusion principle, each energy level contains two electrons i.e each energy level will have two sub energy levels so above equation should be multiplied by 2

$$g^1(E)dE = 2 \frac{\pi}{4} \left(\frac{8ma^2}{h^2} \right)^{\frac{3}{2}} E^{\frac{1}{2}} dE$$

$$= \frac{\pi}{2} \left(\frac{8ma^2}{h^2} \right)^{\frac{3}{2}} E^{\frac{1}{2}} dE$$

$$= \frac{\pi}{2} \left(\frac{8m}{h^2} \right)^{\frac{3}{2}} a^3 E^{\frac{1}{2}} dE$$

Density of states $g(E)dE = \frac{g^1(E)}{V} dE$ If length of energy level is a then its volume

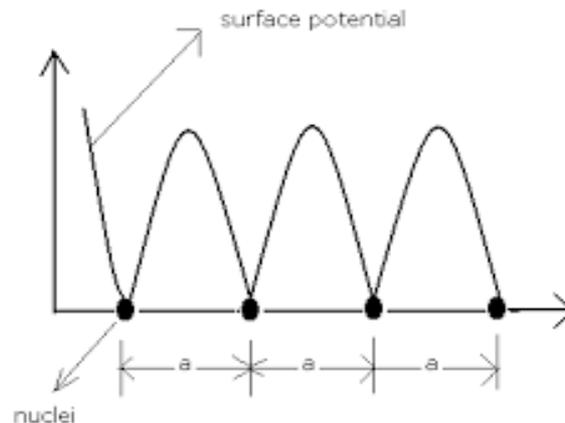
$V = a^3$ So Density of states Can be written as

$$g(E)dE = \frac{\pi}{2} \left(\frac{8m}{h^2} \right)^{\frac{3}{2}} E^{\frac{1}{2}} dE$$

9. Explain Bloch theorem or electrons in a periodic potential.

Electrons in a periodic potential - Bloch theorem:

A crystalline solid consists of a lattice, which is composed of a large number of ion cores at regular intervals, and the conduction electrons that can move freely throughout the lattice. The conduction electrons move inside periodic positive ion cores. Hence instead of considering uniform constant potential as we have done in the electron theory, we have to consider the variation of potential inside the metallic crystal with the periodicity of the lattice as shown fig.



The potential is minimum at the positive ion sites and maximum between the two ions. The one-dimensional Schrödinger equation corresponding to this can be written as

$$\frac{d^2\Psi}{dx^2} + \frac{2m}{\hbar^2} (E - V)\Psi = 0 \quad \text{where } \hbar = h / 2\pi$$

$$\frac{d^2\Psi}{dx^2} + \frac{8\pi^2m}{h^2} (E - V(x))\Psi = 0 \quad \text{-----(1)}$$

The periodic potential $V(x)$ may be defined by means of the lattice constant 'a' as

$$V(x) = V(x + a) \quad \text{----- (2)}$$

Bloch has shown that the one-dimensional solution of the Schrödinger equation is of the form.

$$\Psi(x) = e^{ikx} \cdot U_k(x) \quad \text{-----(3)}$$

In the above equation $U_k(x)$ is called "modulating function". Because free electron wave is modulated by $U_k(x)$ is periodic with the periodicity at the crystal lattice. Let us now consider a linear chain of atoms of length L in one-dimensional case with „N“ number of atoms in the chain. Then

$$U_k(x) = U_k(x + Na) \quad \text{----- (4)}$$

Where 'a' is lattice distance.

$$\begin{aligned} \text{From equation (3) and (4), we have } \Psi(x + Na) &= e^{ik(x + Na)} \cdot U_k(x + Na) \\ &= e^{ikNa} \cdot U_k(x) e^{ikx} \end{aligned}$$

$$\Psi(x + Na) = \Psi(x) \cdot e^{ikNa} \quad \text{----- (5)}$$

This is referred to as Bloch condition. Similarly, the complex conjugate of eq.(5) can be written as

$$\Psi^*(x+Na) = \Psi^*(x) \cdot e^{-ikNa} \quad \text{-----(6)}$$

From eq(5) and eq(6)

$$\Psi(x+Na) \cdot \Psi^*(x+Na) = \Psi(x) \cdot \Psi^*(x)$$

$$|\Psi(x+Na)|^2 = |\Psi(x)|^2 \quad \text{-----(7)}$$

This means that the electron is not localized around one particular atom and the probability of finding the electron is the same throughout the crystal.

Hence in eq(5),

$$e^{ikNa} = 1$$

$$kNa = 2\pi n, \text{ where } n \text{ is an integer}$$

$$k = (2\pi n)/Na$$

$$k = (2\pi n)/L$$

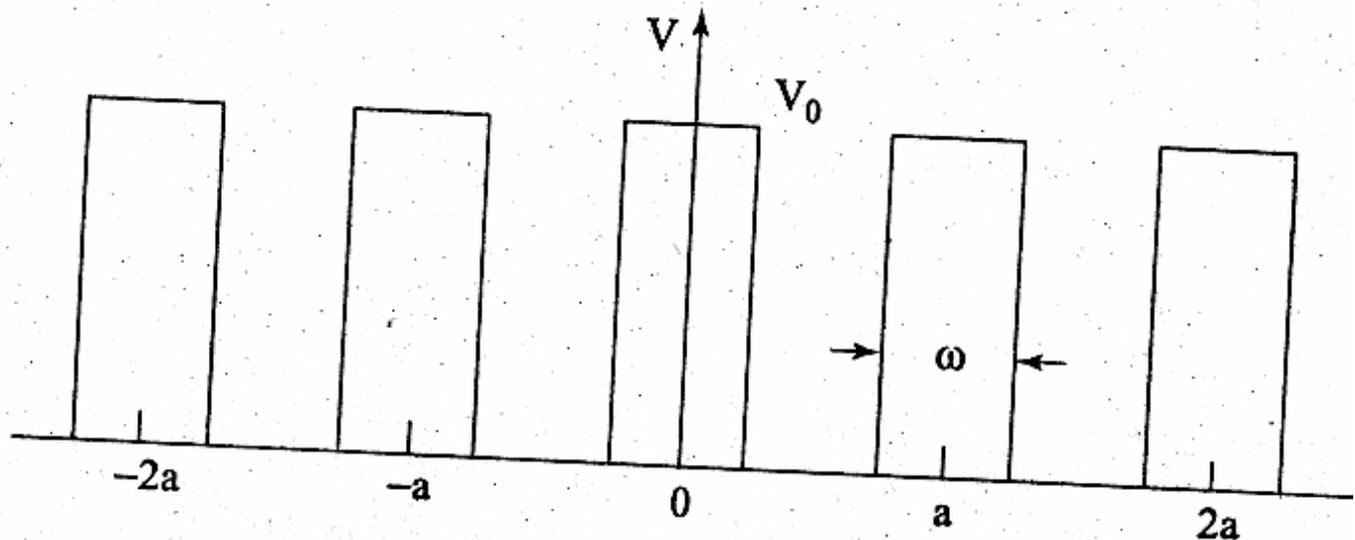
Where L is the length of the chain of atoms and $n = \pm 1, \pm 2, \pm 3, \pm 4, \dots$

When $n = N/2$, we have $k = \pi/a$. This is the edge of the first Brillouin zone.

10. Explain Kronig-Penny model

Kronig - Penny Model:

Kronig - Penny Model proposed a simpler potential in the form of an array of square wells as shown in the fig.



Schrodinger equation for one dimensional periodic potential field denoted by $V(x)$ can be written as

$$\frac{d^2\Psi}{dx^2} + \frac{8\pi^2m}{h^2}(E - V(x))\Psi = 0 \quad \text{-----(1)}$$

According to Bloch theorem the solutions of this equation have the form

$$\Psi(x) = e^{ikx} \cdot U_k(x) \quad \text{-----(2)}$$

Where $U_k(x)$ is periodic with the periodicity of the lattice. That is,

$$U_k(x + a) = U_k(x)$$

As V_0 increases the width of the barrier ω decreases so that the product $V_0\omega$ remains constant. It turns out that solutions are possible only for energies given by the relation

$$\cos ka = \frac{P \sin \alpha a}{\alpha a} + \cos \alpha a \quad \text{-----(3)}$$

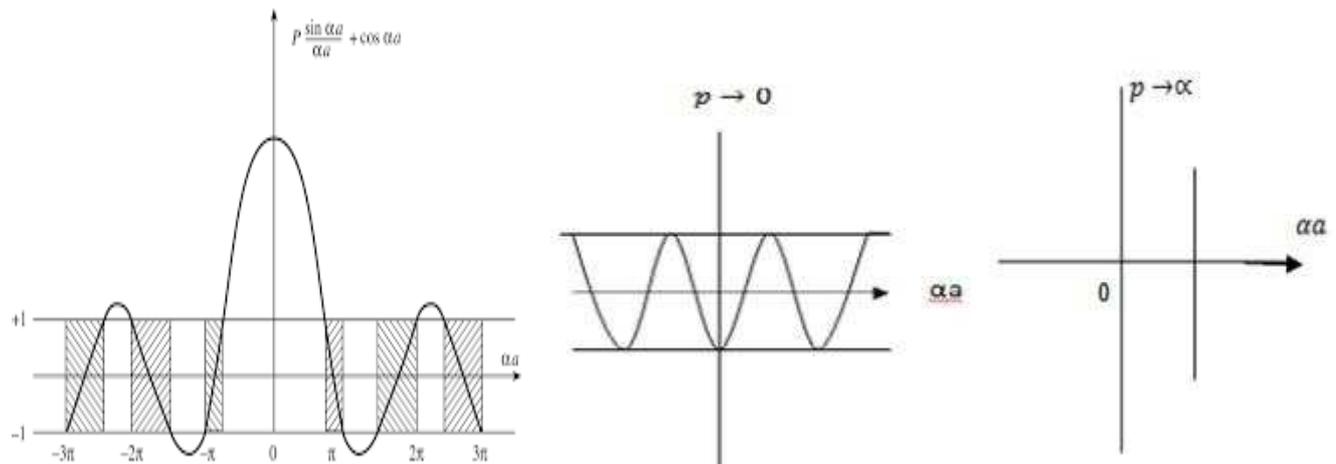
Where $P = \frac{4\pi^2ma}{h^2} V_0\omega$ and $\alpha = \frac{2\pi}{h} \sqrt{2mE}$

P is called scattering power of the barrier and $V_0\omega$ is called barrier strength.

The left hand side of the equation (3) is plotted as a function of ' αa ' for the value of $P = 3\pi/2$ which is shown in fig, the right hand side takes values between -1 to +1 as indicated by the horizontal lines in fig. Therefore the equation (3) is satisfied only for those values of ' ka ' for which left hand side between ± 1 .

From fig, the following conclusions are drawn.

- 1) The energy spectrum of the electron consists of a number of allowed and forbidden energy bands.
- 2) As ' αa ' increases the width of the allowed energy band increases and the width of forbidden band decreases.
- 3) With increasing potential barrier P , the width of an allowed band decreases.
- 4) As $P \rightarrow \infty$, the allowed energy becomes infinitely narrow and the energy spectrum is a line spectrum as shown in fig.
- 5) When $P \rightarrow 0$ then all the electrons are completely free to move in the crystal without any constraints. Hence, no energy level exists that is all the energies are allowed to the electrons as shown in fig. This case supports the classical free electrons theory.

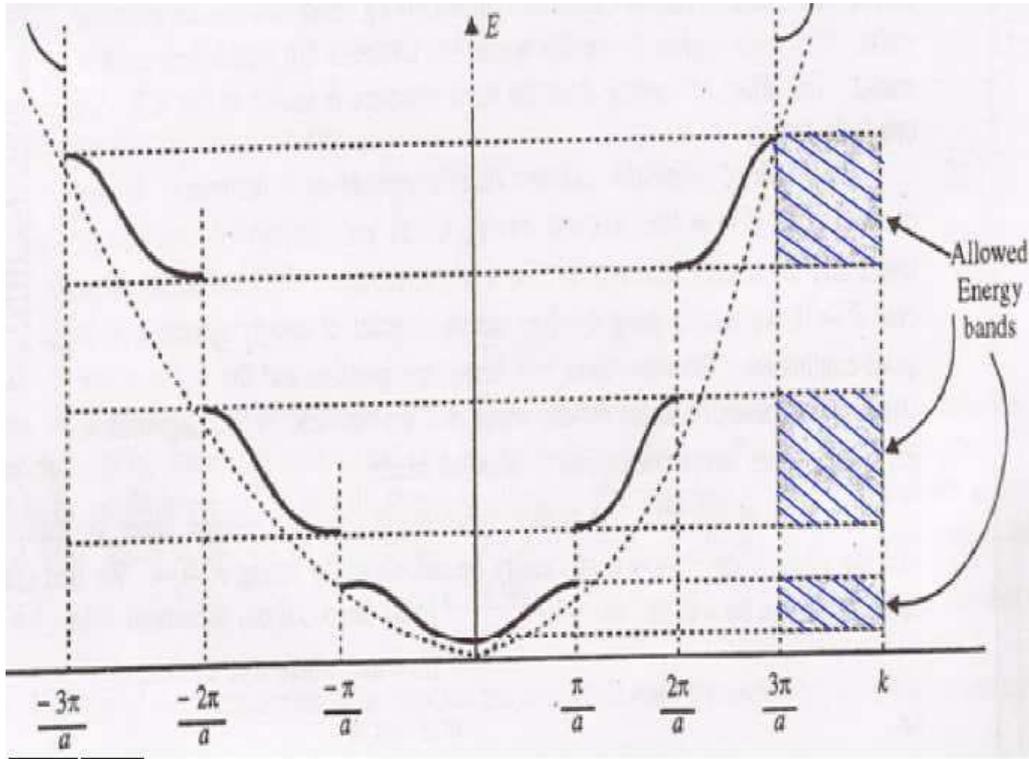


11. Explain Brillouin zones with the help of E-K diagram.

Brillouin Zones (E-K curve):

Using the equation $\cos ka = \frac{P \sin \alpha a}{\alpha a} + \cos \alpha a$

it is possible to plot a curve showing the energy E as a function of K as shown in fig.

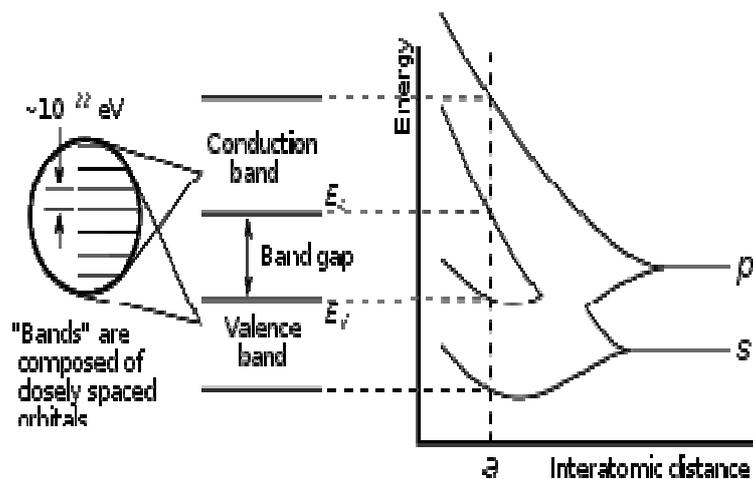


From fig. it is clear that energy of electron is continuously increasing from $K=0$ to $\frac{\pi}{a}$. The left hand side of above equation becomes +1 or -1 for values of $K=\pm\frac{n\pi}{a}$ and hence discontinuity appears in E-K graph, at $K=\pm\frac{n\pi}{a}$. From fig it is seen that energy spectrum of electron consists of allowed regions and forbidden regions. The allowed region extends from $-\frac{\pi}{a}$ to $+\frac{\pi}{a}$. This is known as first Brillouin Zone. After a discontinuity in energy called forbidden gap another allowed region extends from $-\frac{2\pi}{a}$ to $-\frac{\pi}{a}$ and $\frac{\pi}{a}$ to $\frac{2\pi}{a}$. This is known as second Brillouin Zone. Similarly other higher order Brillouin zones can be defined.

12. Explain the origin of energy bands in crystalline solids.

Origin of Energy Bands in Solids:

An isolated atom possesses discrete energies of different electrons. Suppose two isolated atoms are brought to very close proximity, then the electrons in the orbits of two atoms interact with each other. So, that in the combined system, the energies of electrons will not be in the same level but changes and the energies will be slightly lower and larger than the original value. So, at the place of each energy level, a closely spaced two energy levels exists. If ' N ' number of atoms are brought together to form a solid and if these atoms' electrons interact and give ' N ' number of closely spaced energy levels in the place of discrete energy levels, it is known as bands of allowed energies.

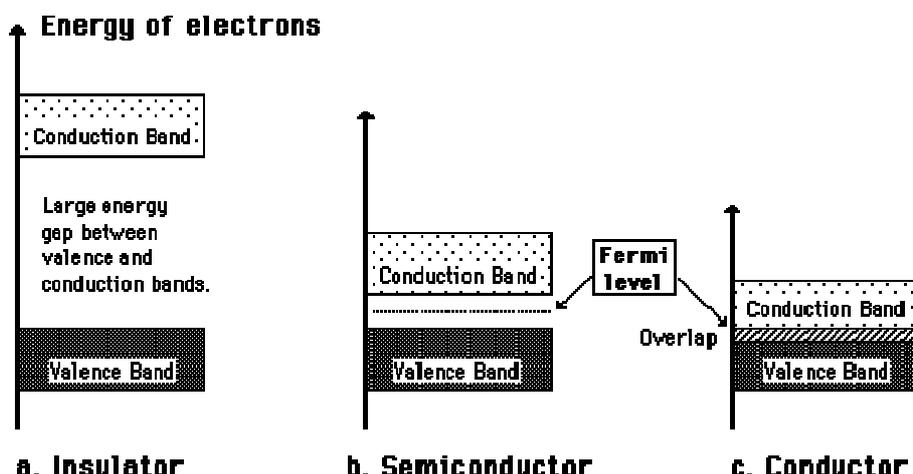


The width of energy band depends on the degree of over lapping of electrons of adjacent atoms and is largest for outermost electrons. The band corresponding to outer most orbits is called conduction band and next inner band is called valence band. The gap between these two allowed bands is called forbidden energy gap or band gap E_g .

13. On the basis of band theory how the crystalline solids are classified into metals, semiconductors and Insulators.

Classification of crystalline solids:

Based on the width of forbidden band, solids are classified into insulators, Semiconductors and conductors.



Conductors: The valence band and conduction bands are overlapped with each other and the energy gap E_g is zero. At room temperatures, free electrons already exist in huge number at conduction band. Hence these solids are good electrical conductors as well as good thermal conductors. The electrical resistivity increases at high temperature by collisions among the free electrons.

Ex: Al, Cu, Ag, Au etc.,

Semiconductors: The valence band and conduction bands are separated with a small energy band gap $E_g \approx 1$ eV. At low temperatures (0 K), free electrons are not available in conduction band. Hence they behave like insulators at low temperatures. The electrical conductivity increases at high temperatures by the transition of free electrons from valence band to conduction band. Thus these solids behave like electrical conductors at high temperatures.

Ex: Silicon $E_g = 1.1$ eV, Germanium $E_g = 0.7$ eV

Insulators: The valence band and conduction bands are separated by a very large energy gap $E_g \geq 3$ eV. At room temperatures conduction band is empty and valence band is full of electrons. Hence these solids are electrical insulators. Even at high temperatures valence electrons are unable to jump in to conduction band.

Ex: Glass, Mica, Ebonite, Rubber etc.,

14. Explain the effective mass of electron and concept of hole.

Effective mass of the electron:

When an electron in a period potential is accelerated by an electric field (or) magnetic field, then the mass of the electron is called effective mass (m^*).

Let an electron of charge 'e' and mass 'm' moving inside a crystal lattice of electric field E. Acceleration $a = eE / m$ is not a constant in the periodic lattice of the crystal. It can be considered that its variation is caused by the variation of electron's mass when it moves in the crystal lattice.

Therefore Acceleration $a = eE / m^*$

Electrical force on the electron $F = m^* a$ -----(1)

Considering the free electron as a wave packet , the group velocity v_g corresponding to the particle's velocity can be written as

$$v_g = \frac{d\omega}{dk} = \frac{d(2\pi\nu)}{dk} = 2\pi \frac{d\nu}{dk} = \left(\frac{2\pi}{h}\right) \frac{d(h\nu)}{dk} = \frac{1}{h} \frac{dE}{dk} \text{ -----(2)}$$

where, the energy $E = hu$ and $\hbar = \frac{h}{2\pi}$

$$\text{Acceleration } a = \frac{dv_g}{dt} = \frac{d\left(\frac{1}{h} \frac{dE}{dk}\right)}{dt} = \frac{1}{h} \frac{d^2E}{dk \cdot dt} = \frac{1}{h} \left(\frac{d^2E}{dk \cdot dk}\right) \left(\frac{dk}{dt}\right) = \frac{1}{h} \left(\frac{d^2E}{dk^2}\right) \left(\frac{dk}{dt}\right)$$

$$\text{Since } \hbar k = p \text{ and } \frac{dP}{dt} = F, \quad \frac{dk}{dt} = \frac{F}{\hbar}$$

$$\therefore a = \frac{1}{\hbar^2} \left(\frac{d^2E}{dk^2}\right) F$$

$$F = \left[\frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)} \right] a \text{ -----(3)}$$

Comparing equations (1) and (3) we get

$$m^* = \left[\frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)} \right] \text{-----(4)}$$

This equation indicates that the effective mass is determined by $\frac{d^2E}{dk^2}$.

a. Variation of E with K: Variation of E with K corresponds to first Brillouin Zone. Using this type of variation velocity can be calculated.

b. Variation of V with K: For $K=0$ velocity is zero and as k value increases V also increases reaching its maximum value at K_0 known as point of inflexion. Beyond K_0 velocity decreases and reaches to 0 at $K=\pi/a$.

c. Variation of m^* with K: Near $K=0$, $m^*=m$. As K value increases m^* also increases reaching its maximum value at K_0 . Above K_0 , m^* is negative and decreases as K tends to π/a . Beyond K_0 velocity decreases and hence retarding force acts on electron and it behaves as positively charged particle referred to as hole.

d. Variation of f_k : Degree of freedom of an electron is defined as $f_k = m/m^*$. f_k is measure of the extent to which the electron is free in a state K . If m^* is large f_k is small, particle behaves as heavy particle. If $f_k=1$, electron behaves as free electron.

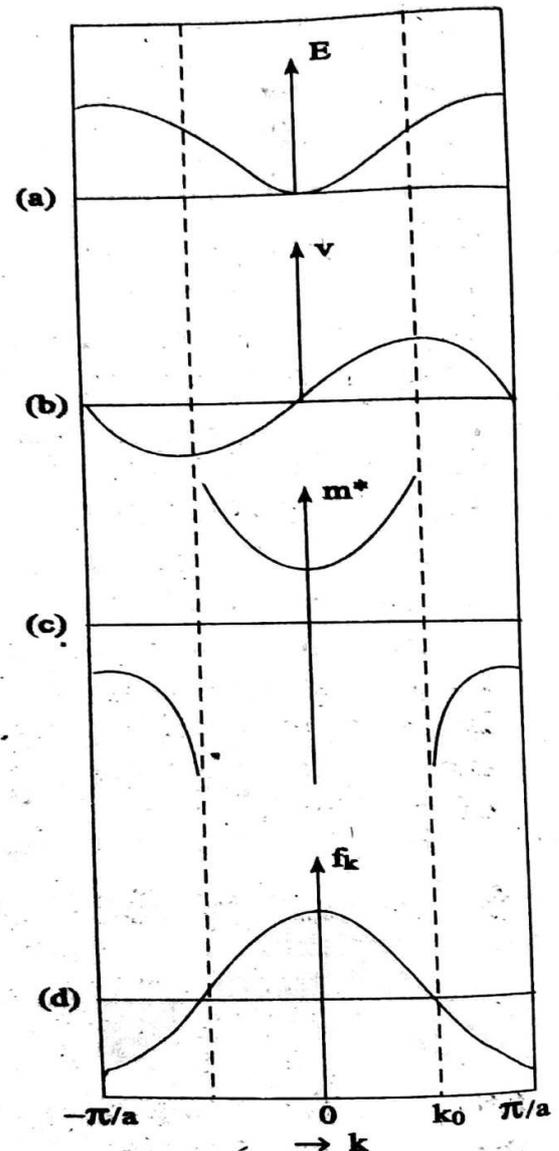


Fig. 1 Energy, effective mass and f_k as a function of K .

Assignment questions

1. What are the merits and demerits of classical free electron theory?
2. What are the assumptions of quantum free electron theory? What are the merits and demerits?
3. Derive an expression for electrical conductivity on the basis of quantum free electron theory.
4. What is Fermi level? Explain the Fermi-Dirac distribution function of electrons. Illustrate graphically the effect of temperature on the distribution.
5. What is Bloch theorem? Explain.
6. Discuss the Kronig-Penney model for the motion of an electron in a periodic potential.
7. Explain the concept of effective mass of electron and hole.
8. Explain the origin of energy bands in solids. On the basis of band theory how the crystalline solids are classified into metals, semiconductors and Insulators?

Problems

1. Evaluate the Fermi function for an energy KT above Fermi energy.
2. At what temperature we can expect 10% probability that the electrons in silver have an energy which is 1% above the Fermi energy. The Fermi energy of silver is 5.5 eV.
3. Calculate the Fermi energy in eV for silver at 0 K. The number of conduction electrons in silver is $5.863 \times 10^{28}/\text{m}^3$
4. Fermi level for potassium is 2.1eV. Calculate the velocity of the electron at Fermi level.
5. In a solid consider the energy level lying 0.01eV below the Fermi level. What is the probability of this level not being occupied by an electron?

16. From the E-K curve at $k=0$ the velocity of electrons is
a) maximum b) minimum **c) zero** d) moderate
17. In an allowed band the velocity of electron is zero at
a) top **b) bottom and top** c) middle d) bottom
18. Band theory of solids is mainly due to
a) overlapping of all the orbits b) overlapping of outermost orbits
c) closely packed atoms **d) both b and c**
19. At absolute zero the semiconductors have the band structure similar to
a) conductors **b) insulators** c) semiconductors d) superconductor
20. First Brillouin zone corresponds to k value extending from
a) 0 to $\frac{\pi}{a}$ b) $\frac{\pi}{a}$ to $\frac{2\pi}{a}$ c) $-\frac{2\pi}{a}$ to $\frac{2\pi}{a}$ **d) $-\frac{\pi}{a}$ to $\frac{\pi}{a}$**