Code No: RT21024 **R13** SET - 1

II B. Tech I Semester Regular Examinations, Dec - 2015
COMPLEX VARIABLES AND STATISTICAL METHODS
(Electrical and Electronics Engineering)

Time: 3 hours Max. Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B)**
2. Answer **ALL** the question in **Part-A**
3. Answer any **THREE** Questions from **Part-B**

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Note :- Statistical tables are required

PART –A

1. a) Is $f(z)=z^3$ analytic? (4M)
   b) State Cauchy's Integral Theorem. (3M)
   c) Find the residues at the singular points of $\dfrac{9z+i}{z(z^2+1)}$. (4M)
   d) Find the fixed points of the transformation $w=(z-i)^2$. (4M)
   e) Define Population and Sample with examples. (3M)
   f) Define one-tailed and two-tailed tests. (4M)

PART –B

2. a) Show that for $f(z)=\begin{cases}\dfrac{(x^3-y^3)+i(x^3+y^3)}{x^2+y^2}, & z\neq 0\\ 0 & ,z=0\end{cases}$ the Cauchy-Riemann equations are satisfied at the origin but the derivative of $f(z)$ at origin does not exist. (10M)
   b) Find the analytic function $f(z)=u+iv$ where $v=e^x\sin y$. (6M)

3. a) Evaluate $\int_0^{3+i} z^2\,dz$ along real axis from 0 to 3 and then vertically to $3+i$. (8M)
   b) Find the Laurent series of $f(z)=\dfrac{1}{z^2-4z+3}$, for $1<|z|<3$. (8M)

4. a) Determine the poles of the function $f(z)=\dfrac{z+1}{z^2(z-2)}$ and the residue at each pole. (8M)
   b) Use residue theorem to evaluate $\displaystyle\int_0^{2\pi}\frac{d\theta}{5-3\sin\theta}$. (8M)
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5. a) Determine the image of the region $|z-3|=5$ under the transformation $w=\frac{1}{z}$. (8M)

   b) Determine the bilinear transformation that maps the points $z_1=0$, $z_2=2i$, $z_3=-2i$ into the points $w_1=-1$, $w_2=0$, $w_3=\infty$ respectively. (8M)

6. a) Determine the probability that $\overline{X}$ will be between 66.8 and 68.3 if a random sample of size 25 is taken from an infinite population having the mean $\mu=68$ *and* $\sigma=3$. (8M)

   b) The average zinc concentration recovered from a sample of zinc measurements in 36 different locations is found to be 2.6 grams per milliliter. Find a 95% confidence intervals for the mean zinc concentration in the river. Assume that the population standard deviation is 0.3. (8M)

7. a) A storekeeper wanted to buy a large quantity of bulbs from two brands A and B respectively. He bought 100 bulbs from each brand A and B and found by testing brand A had mean life time of 1120 hrs and the S.D of 75 hrs and brand B had mean life time 1062 hrs and S.D of 82 hrs. Examine whether the difference of means is significant. Use a 0.01 level of significance. (8M)

   b) An urban community would like to show that the incidence of breast cancer is higher than in a nearby rural area. If it is found that 20 of 200 adult women in the urban community have breast cancer and 10 of 150 adult women in the rural community have breast cancer, can we conclude at the 0.01 level of significance that breast cancer is more prevalent in the urban community? (8M)

Code No: RT21024 **R13** SET - 2

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2. Answer **ALL** the question in **Part-A**
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Note :- Statistical tables are required

PART –A

1. a) Is $f(z)=z^2$ analytic? (4M)
   b) State Residue theorem. (3M)
   c) Find the residues at the singular points of $\frac{4-z}{z^2-z}$. (4M)
   d) Find the fixed points of the transformation $w=\frac{z+i}{z-i}$. (4M)
   e) Find the value of the finite population correction factor for n = 10 and N = 1000. (3M)
   f) Define Type-I and Type-II errors in testing of hypothesis. (4M)

PART –B

2. a) Show that for $f(z)=\begin{cases} \frac{xy^2(x+iy)}{x^2+y^4}, & z\neq 0 \\ 0 & , z=0 \end{cases}$ the Cauchy-Riemann equations are satisfied at the origin but the derivative of $f(z)$ at origin does not exist. (10M)
   b) Find the analytic function $f(z)=u+iv$ where $u=\sin x\cosh y$. (6M)

3. a) Integrate $f(z)=x^2+ixy$ from $A(1,1)\, to\, B(2,8)$ along the curve $x=t$, $y=t^3$. (8M)
   b) Use Cauchy's integral formula to evaluate $\oint_C \frac{e^{2z}}{(z-1)(z-2)}dz$ where C is the circle $|z|=3$. (8M)

4. a) Determine the poles of the function $f(z)=\frac{z^2+1}{z^2-z}$ and the residue at each pole. (8M)
   b) Use residue theorem to evaluate $\int_{-\infty}^{\infty}\frac{dx}{(x^2+1)(x^2+9)}$. (8M)
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5. a) Find and sketch the image of the region $-1 \le x \le 0$, $0 \le y \le \frac{\pi}{2}$ under $w = e^z$. (8M)

b) Determine the bilinear transformation that maps the points $z_1 = 0$, $z_2 = 1$, $z_3 = 2$ into the points $w_1 = 1$, $w_2 = \frac{1}{2}$, $w_3 = \frac{1}{3}$ respectively. (8M)

6. a) Find $P(\bar{X} > 66.75)$ if a random sample of size 36 is drawn from an infinite population with mean $\mu = 63 \, and \, \sigma = 9$. (8M)

b) Determine a 95% confidence interval for the mean of a normal distribution with variance $\sigma^2 = 0.25$, using a sample of $n = 100$ values with mean $\bar{x} = 212.3$. (8M)

7. a) Two independent samples of size 9 and 8 had the following values of the variables : (8M)

Sample I	17	27	18	25	27	29	27	23	17
Sample II	16	16	20	16	20	17	15	21	

Do the estimates of the population variance differ significantly? Use a 0.05 level of significance.

b) In a study to estimate the proportion of residents in a certain city and its suburbs who favor the construction of a nuclear power plant, it is found that 63 of 100 urban residents favor the construction while only 59 of 125 suburban residents are in favor. Is there a significant difference between the proportion of urban and suburban residents who favor construction of the nuclear plant? Use a 0.05 level of significance. (8M)

Code No: RT21024 **R13** SET - 3

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2. Answer **ALL** the question in **Part-A**
3. Answer any **THREE** Questions from **Part-B**

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Note :- Statistical tables are required

PART –A

1. a) Check analyticity by using Cauchys-Riemann Equations for $f(z) = z\,\overline{z}$. (4M)
   b) State Laurent's theorem. (3M)
   c) Find the residues at the singular points of $\dfrac{4}{(z^2+1)}$. (4M)
   d) Find the invariant points of the transformation $w = \dfrac{2iz-1}{z+2i}$. (4M)
   e) Explain interval estimators of mean. (3M)
   f) Write procedure for test of hypothesis concerning one mean for large sample. (4M)

PART –B

2. a) Show that $f(z) = \begin{cases} \dfrac{x^2 y^5 (x+iy)}{x^4 + y^{10}}, & z \neq 0 \\ 0 & , z = 0 \end{cases}$ is not analytic at $z = 0$, although the Cauchy-Riemann equations are satisfied at the origin. (10M)
   b) Find the analytic function $f(z) = u + iv$ where $u = e^x \cos y$. (6M)

3. a) Evaluate $\int_0^{1+i} (x^2 + iy)\,dz$ along the paths $y = x$ and $y = x^2$. (8M)
   b) Find Taylor's expansion of $f(z) = \dfrac{2z^3+1}{z^2+z}$ about the point $z = i$. (8M)

4. a) Determine the poles of the function $f(z) = \dfrac{z^2}{(z-1)^2(z+2)}$ and the residue at each pole. (8M)
   b) Use residue theorem to evaluate $\int_{-\infty}^{\infty} \dfrac{x^2}{(x^2+1)(x^2+4)}\,dx$. (8M)
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5. a) Find and sketch the image of the region $-0.5 \le x \le 0.5$, $\frac{3\pi}{4} \le y \le \frac{5\pi}{4}$ under $w = e^z$. (8M)

   b) Determine the bilinear transformation that maps the points $z_1 = 0$, $z_2 = 1$, $z_3 = \infty$ into the points $w_1 = -1$, $w_2 = -i$, $w_3 = 1$ respectively. (8M)

6. a) Determine the probability that $\overline{X}$ will be between 22.39 and 22.41 if a random sample of size 36 is taken from an infinite population having the mean $\mu = 22.4$ *and* $\sigma = 0.048$. (8M)

   b) The contents of 7 similar containers of sulfuric acid are 9.8, 10.2, 10.4, 9.8, 10.0, 10.2, and 9.6 liters. Find a 95% confidence interval for the mean of all such containers, assuming an approximate normal distribution. (8M)

7. a) In a random sample of 100 tube lights produced by company A, the mean life time of tube light is 1190 hours with standard deviation of 90 hours. Also in a random sample of 75 tube lights from company B the mean life time is 1230 hours with standard deviation of 120 hours. Is there a difference between the mean lifetimes of the two brands of tube lights at a significance level of 0.05? (8M)

   b) Two samples of sodium vapor bulbs were tested for length of life and the following results were returned : (8M)

	Size	Sample mean	Sample S.D.
Type I	8	1234 hrs	36 hrs
Type II	7	1036 hrs	40 hrs

   Is the difference in the means significant to generalize that type I is superior to type II regarding length of life? Use a 0.05 level of significance.

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2. Answer **ALL** the question in **Part-A**
3. Answer any **THREE** Questions from **Part-B**

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Note :- Statistical tables are required

PART –A

1. a) Check analyticity by using Cauchys-Riemann Equations for $f(z) = e^x(\cos y + i\sin y)$. (4M)
   b) State Taylor's theorem. (3M)
   c) Find the residues at the singular points of $\tan z$. (4M)
   d) Find the invariant points of the transformation $w = \dfrac{-3iz-5}{z+i}$. (4M)
   e) Define point estimator and unbiased estimator. (3M)
   f) Write χ^2 statistic for test of goodness of fit. (4M)

PART -B

2. a) Show that $f(z) = \sqrt{|xy|}$ is not analytic at $z = 0$, although the Cauchy-Riemann equations are satisfied at the origin. (10M)
   b) Find the analytic function $f(z) = u + iv$ where $u = x^3 - 3xy^2$. (6M)

3. a) Integrate $f(z) = x^2 + ixy$ from $A(1,1)\,to\,B(2,4)$ along the curve $x = t$, $y = t^2$. (8M)
   b) Use Cauchy's integral formula to evaluate $\oint_C \dfrac{e^z}{(z^2+\pi^2)^2}\,dz$ where C is the circle $|z| = 4$. (8M)

4. a) Find the residue of $f(z) = \dfrac{ze^z}{(z-1)^3}$ at its pole. (8M)
   b) Use residue theorem to evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$. (8M)
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5. a) Determine and Plot the image of the region $2<|z|<3$ and $|\arg z|<\frac{\pi}{4}$ under $w=z^2$. (8M)

   b) Determine the bilinear transformation that maps the points $z_1=-1$, $z_2=i$, $z_3=1$ into the points $w_1=0$, $w_2=i$, $w_3=\infty$ respectively. (8M)

6. a) Determine the probability that $\overline{X}$ will be between 75 and 78 if a random sample of size 100 is taken from an infinite population having the mean $\mu=76 \, and \, \sigma^2=256$. (8M)

   b) A random sample of 8 envelopes is taken from the letter box of a post office and their weights in grams are found to be: 12.1, 11.9, 12.4, 12.3, 11.5, 11.6, 12.1, and 12.4. Find 95% confidence limits for the mean weight of the envelopes received at that post office. (8M)

7. a) A manufacturer claims that the average tensile strength of thread A exceed the average tensile strength of thread B by at least 12 kilograms. To test his claim, 50 pieces of each type of thread are tested under similar conditions. Type A thread had an average tensile strength of 86.7 kilograms with known standard deviation of $\sigma_A=6.28$ kilograms, while type B thread had an average tensile strength of 77.8 kilograms with known standard deviation of $\sigma_B=5.61$ kilograms. Test the manufacturers claim at 0.01 level of significance. (8M)

   b) In two independent samples of sizes 8 and 10 the sum of squares of deviations of the sample values from the respective sample means were 84.4 and 102.6. Test whether the difference of variances of the populations is significant or not. Use a 0.05 level of significance. (8M)