# BENDING STRESSES IN BEAMS

The stresses produced due to constant Bending Moment (with zero Shear Force or pure bending) are known as Bending stresses.

## 1. Simple bending theory

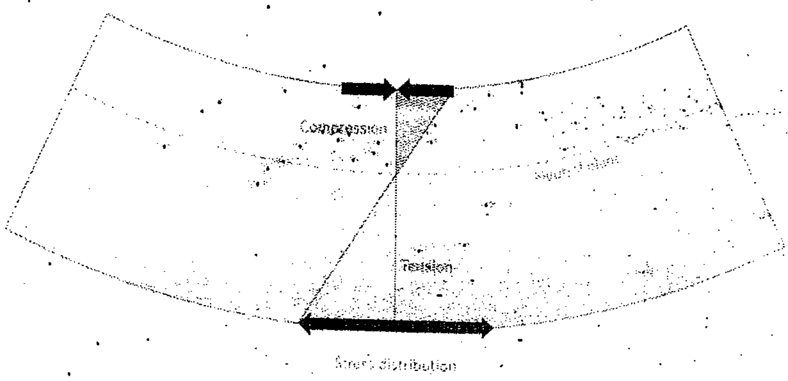
Bending stress is resistance offered by the internal stresses to the bending caused by bending moment at a section. The process of bending stops, when every cross-section sets up full resistance to the bending moment. It only couples are applied to the ends of the beam and no forces acted on the bar, the bending is termed as **pure bending**.
For example, the portion of beam between the two downward forces is subjected to pure bending. The bending produced by forces that do not form couples is called ordinary bending. A beam subjected to pure bending has only normal stresses with no shearing set up in it. A beam subjected to ordinary bending has both normal and shearing stresses acting within it.
Now, it is a length of beam is acted upon by a constant bending moment (zero shearing force) as shown below in figure. The bending moment will be same at all point along the bar i.e., a condition of pure bending.

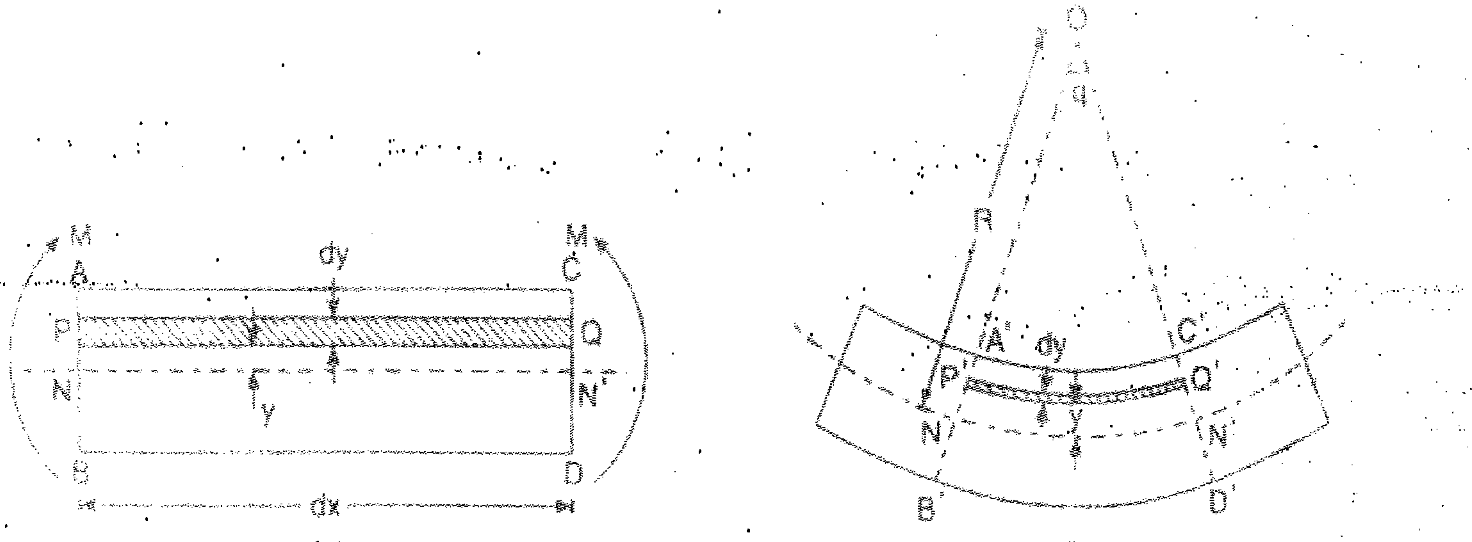
## 2. Assumptions in Theory of Bending

In deriving the relations between the bending moments and flexural (bending) stresses the following assumptions are made.
1. The material of the beam is isotropic and homogeneous and follows Hooke's law and has the same value of Young's Modulus in tension and compression.
2. Transverse sections of the beam that were plane before bending remains plane even after bending also.
3. The beam is initially straight and subjected to Pure bending, all longitudinal fillimants bends in an arc of a circle with common center of curvature
4. The radius of curvature is large compared to the dimension of the cross-section.
5. Each layer is independent to enlarge or contract.
6. The stresses are purely longitudinal and local effects of point loads are neglected.

## 3. Flexural / bending formula

To determine the distribution of bending stress in the beam, let us cut the beam by a plane passing through it in a direction perpendicular to the geometric axis of the bar. Let the sections be AB and CD normal to the axis of the beam. Due to the action of bending moment, the beam as a whole bend as shown in figure.






(a) (b)

Since, we are considering a small length of **dx** of the beam, the curvature of the beam could be considered to be circular. Because of it, top layer of the beam **AC** suffers compression and lowest layer **BD** suffers extension.

The amount of compression or extension depends upon the position of the layer with respect to NN′. The layer NN′ is neither compressed nor stretched and has zero stress. It is called neutral surfaces of neutral plane.

From figure, we have

Length of layer PQ after applying moment = P′Q′ = $(R - y)\varnothing$

Decrease in length of PQ = PQ – P′Q′

Strain '$\varepsilon$' = Change in length/ original length (PQ – P′Q′) / PQ

$$= [R - (R - y)\varnothing] / R\varnothing = [R - R + y] / R = y/R$$

Since from figure NN′ = PQ = $R\varnothing$

**Therefore the strain in the layer $\varepsilon = y / R$**

**But we know** $E = \sigma/\varepsilon \Rightarrow \sigma = \varepsilon.E$

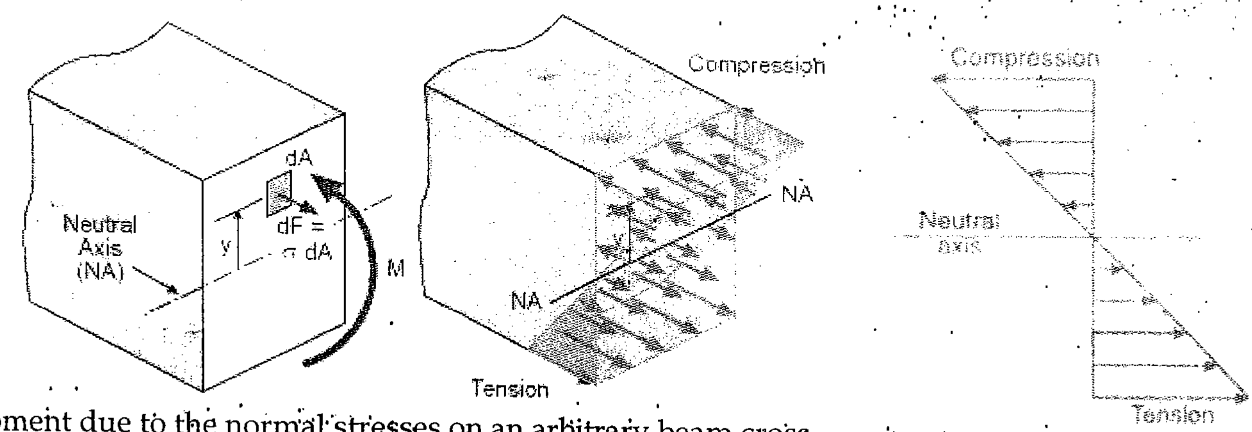
$$\Rightarrow \sigma = [y/R].E = (E/R)\,y$$

$$\sigma / y = E / R \qquad \text{----(1)}$$

Where, E is the modulas of elasticity.
R is radius of curvature of neutral axis,
y distance of layer under consideration from neutral axis

Thus, **Since, E/R is constant, the stress is proportional to the distance from neutral axis as shown in** fig. the strain produced in a layer is also proportional to its distance from neutral axis.
Thus, of the purpose of economy and weight reduction the material should be concentrated as much as possible at the greatest distance from neutral axis. Hence, I sections should be preferred.

## 3.1 Moment of Resistance




The moment due to the normal stresses on an arbitrary beam cross section and equating it to the applied internal moment. This is the same as applying the moment equilibrium equation about the neutral axis (NA).

$$\sum M_{NA} = 0$$

$$\int y(-dF) = M$$

$$-\int y\,\sigma dA = M$$

For a positive moment, the top stresses will be in compression (negative stress) and the bottom stresses will be in tension (positive stress) and thus the negative sign in the equation. This equation can be changed by using equation (1) ,

$$\sigma / y = E / R$$

$$\frac{E}{\rho}\int y^2\, dA = M$$

It is interesting to note that the integral is the area moment of inertia, I, or the second moment of the area. Using the area moment of inertia gives

$$E\, I/R = M$$

$$E/R = M/I \qquad \text{---- (2)}$$

## 4. Flexure/ Bending Formula :

By taking equations one and two, the bending equation is given by,

$$M/I = f/y = E/R$$

Where **M** = Bending Moment
**I** = Moment of inertia about Neutral axis (N.A.), **f** = Bending stress (σ)
**y** = Distance of the fiber from N.A. **R** = Radius of Curvature, **E** = Young's Modulus
This equation may be remember as

**" May I flow you Every Rule "- M/I = f/y = E/R**

## 5. Location of Neutral Axis

The neutral axis is always located at the centroid (geometric center) of the cross section.

## 6. Section modulus

It is a geometric property for a given cross-section used in the design of beams or flexural members. The elastic section modulus is defined as

$$Z = I / y_{max},$$

where I is the second moment of area (or moment of inertia) and $y_{max}$ is the distance from the Neutral axis to any extreme fibre.

$$M/I = f/y$$

$$M = f\,(I/y)$$

But the maximum stress $f_{max}$ will occurs when the distance y is maximum – y max

The maxium moment of resistance $M = f_{max}\,(I/y_{max})$

$$M = f_{max}.Z$$

**Note:** A given value of allowable bending stress the moment of resistance depends upon the value of section modulus.

The strength of a section means the moment of resistance offered by the section.

### 6.1 Section Modulus Of Various Sections

- Rectangle section – $Z = b\,d^2/6$
- Hollow Rectangle section - $Z = (BD^2 - bd^2)/6$
- Solid circular section $Z = \pi d^3/32$
- Hollow circular Section $Z = \pi(D^4 - d^4)/32D$
- Other sections we calculate the Z by using Moment of Inertia of section about its Neutral axis ($I_{NA}$) and maximum fiber distance ($y_{max}$) form N.A

UNIT-3 S.M. – I ASSINMENT –III

**1. Write assumptions in simple bending theory and derive simple bending equations**

**2. A beam of I section has overall dimensions of 350×150 mm. The thickness of flange and web are 10 mm. Calculate the maximum bending stress across a section and draw the stress distribution, if the beam carries a udl of 12 kN/m for span of 5 m.**

**3. Define section modulus and derive section modulus for rectangular and circular sections.**

**4. Compare the strength of solid circular, rectangular and I sections of equal weight**

**5. A cantilever beam of 230mm Wide and 350mm deep is 4m long. It is loaded with udl of 4KN/m over the entire length and a point load of 5KN is placed at the free end. Find the maximum bending stresses produced in the beam and draw stress distribution diagram.**

**6. The maximum bending moment acting in an I – section (shown in fig.) is 40kN-m. Find the maximum bending stress developed in the section and draw shear stress distribution diagram**