UNIT-1

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1. Introduction

When a force is applied on a body it suffers a change in shape, that is, it deforms. A force to resist the deformation is also set up simultaneously within the body and it increases as the deformation continues. The process of deformation stops when the internal resisting force equals the externally applied force. If the body is unable to put up full resistance to external action, the process of deformation continues until failure takes place.

The deformation of a body under external action and accompanying resistance to deform are referred to by the terms strain and stress respectively.

2. Stress

Stress is defined as the internal resistance per unit area set up by a body when it is deformed.

It is measured in N/m^2 or Pascal (Pa). 1 Pa = $1N/m^2$,
1MPa = 10^6 N/m^2 = $1N/mm^2$.

2.1 Basic Types of Stresses

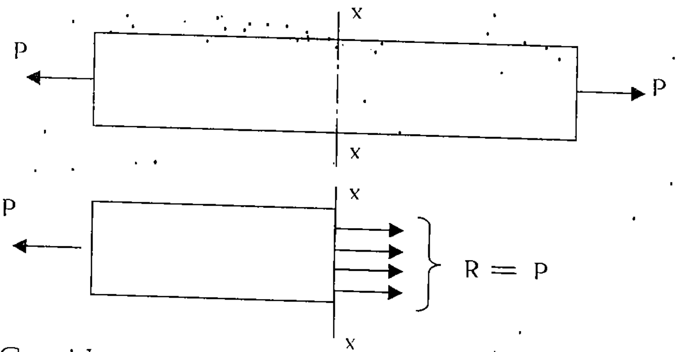
Basically two types of stresses can be identified. These are related to the nature of the deforming force applied on the body.

- The stress is Normal to cross section they is called normal stress or Direct stress.
- The stress acting tangential to the cross section is called shear stress.

2.2. Normal or Direct Stresses

When the stress acts at a section or normal to the plane of the section, it is called a normal stress or a direct stress. It is a term used to mean both the tensile stress and the compressive stress.

2.2.1. Tensile Stress



Consider a uniform bar of cross sectional area A subjected to an axial tensile force P. The stress at any section x-x normal to the line of action of the tensile force P is specifically called tensile stress. Since internal resistance R at x-x is equal to the applied force P, we have, Normal stress(tension) σ_t = (internal resistance at x-x)/(resisting area at x-x)

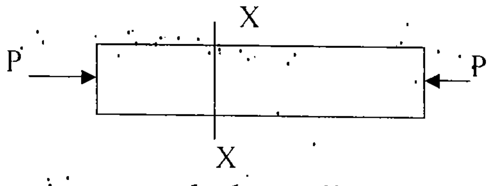
σ_t =R/A =P/A.

Under tensile stress the bar suffers stretching or elongation.

2.1.2. Compressive Stress

If the bar is subjected to axial compression instead of axial tension, the stress developed at x-x is specifically called compressive stress p_c.

Normal stress (compression) σ_c =R/A = P/A.



Under compressive stress the bar suffers shortening.

3. Strains

The deformation for one unit length is called strain.

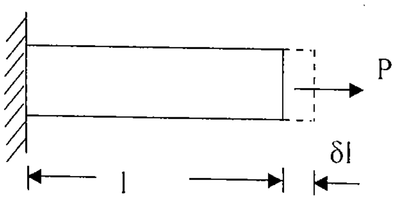
Strain is a the ratio of change in dimension to original dimension of a body when it is deformed. It is a dimensionless quantity as it is a ratio between two quantities of same dimension.

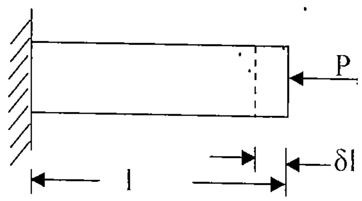
3.1. Linear Strain

Linear strain or longitudinal strain of a deformed body is defined as the ratio of the change in length of the body due to the deformation to its original length in the direction of the force. If l is the original length and δl the change in length occurred due to the deformation,

The linear strain ε induced is given by

$$\varepsilon = \delta l / l.$$





Linear strain may be a tensile strain , ε_t or a compressive strain ε_c according as δl refers to an increase in length or a decrease in length of the body.

If we consider tensile strain as +ve then the compressive strain should be considered as -ve, as these are opposite in nature.

3.2. Lateral Strain

The direction perpendicular to the force is called lateral direction.

Lateral strain of a deformed body is defined as the ratio of the change in unit lateral dimension.

4. Hooke's Law

Hooke's law states that stress is proportional to strain upto elastic limit. If I is the stress induced in a material and e the corresponding strain, then according to Hooke's law, $\sigma / \varepsilon = E$, a constant.

This constant E is called the modulus of elasticity or Young's Modulus, (named after the English scientist Thomas Young).

4.1 Modulus of Elasticity or Young's Modulus (E)

The stress which is required to produce one unit strain is called Young's Modulus.

Modulus of Elasticity is the ratio of direct stress to corresponding linear strain within elastic limit. If σ is any direct stress below the elastic limit and ε the corresponding linear strain, then $E = \sigma / \varepsilon$.

4.3 Elogation of Bars of uniform cross section

Consider a bar of length l and Cross sectional area A. Let P be the axial pull on the bar, σ the stresss induced, ε the strain in the bar and δl is the elongation.

Then $\sigma = P/A$

$\varepsilon = \sigma/E = P/(AE)$ ------------(1)

$\varepsilon = \delta l/l$ ------------(2)

equating (1) and (2)

$$\delta l = Pl/(AE)$$

4.4. Temperature or Thermal stresses.

The stresses induced in a body due to change in temperature are known as thermal stresses.

Change inlength due to temp. change $\delta t = \alpha . T . L$

Thermal strain and thermal stress is given by

Thermal strain, $\varepsilon t = \alpha . T$ and

Thermal stress $\sigma = \alpha . T . E$

where α = Co-efficient of linear expansion, T = Rise or fall of temperature
E = Young's modulus, L = length of member.

In composite material section , Due to the temperature changes , it should satisfies the below relation;

$$\varepsilon_1 + \varepsilon_2 = (\alpha_2 - \alpha_1) T$$

4.4. Poisson's Ratio

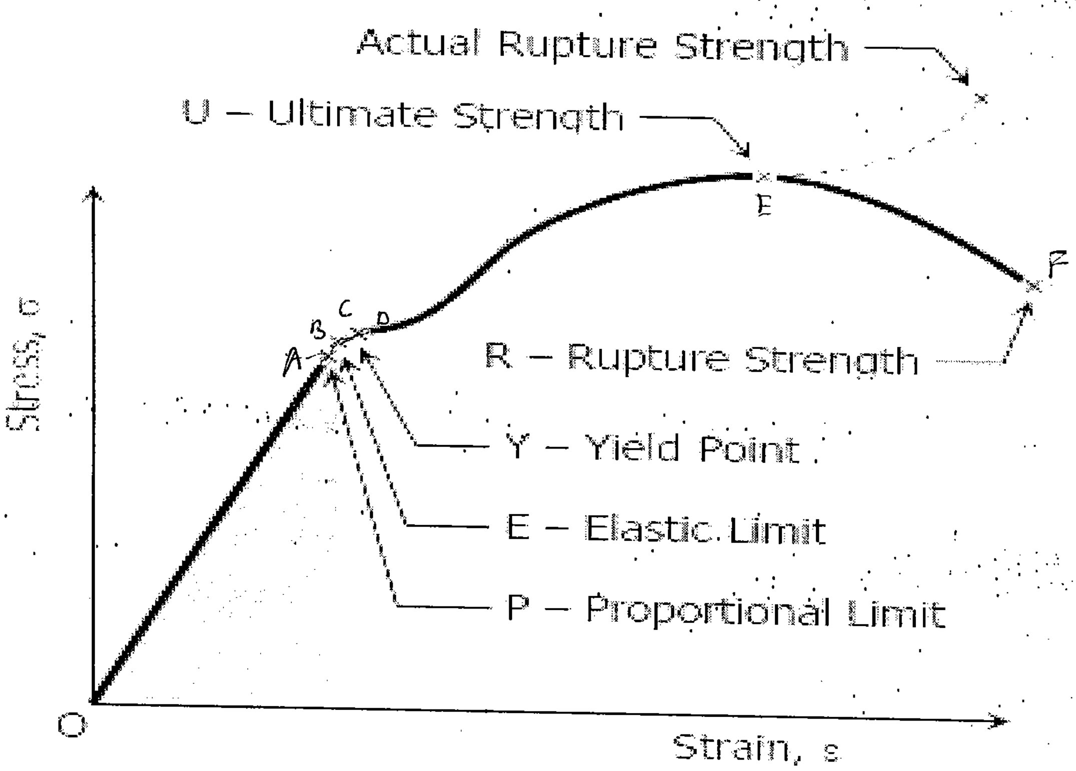
Any direct stress is accompanied by a strain in its own direction and called linear strain (longitudinal strain) and an opposite kind strain in every direction at right angles to it is called lateral strain. This lateral strain bears a constant ratio with the linear strain. This ratio is called the Poisson's ratio and is denoted by μ.

Poisson's Ratio = Lateral Strain / Linear Strain.

Value of the Poisson's ratio for most materials lies between 0.25 and 0.33.

5.4. Stress and Strain Diagram of Mild steel Bar

A standard mild steel specimen is subjected to a gradually increasing pull by Universal Testing Machine. The stress-strain curve obtained is as shown below.



Elasticity and Elastic Limit

Elasticity of a body is the property of the body by virtue of which the body regains its original size and shape when the deformation force is removed. Most materials are elastic in nature to a lesser or greater extend, even though perfectly elastic materials are very rare.

The maximum stress upto which a material can exhibit the property of elasticity is called the elastic limit. If the deformation forces applied causes the stress in the material to exceed the elastic limit, there will be a permanent set in it. That is the body will not regain its original shape and size even after the removal of the deforming force completely. There will be some residual strain left in it.

Yield stress

When a specimen is loaded beyond the elastic limit the stress increases and reach a point at which the material starts yielding this stress is called yield stress.

Ultimate stress

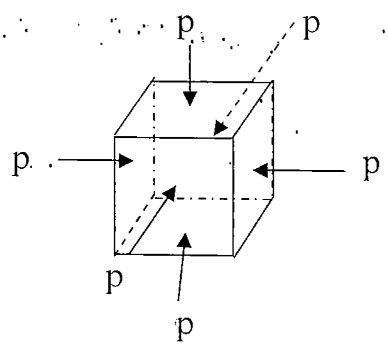
Ultimate load is defined as maximum load which can be placed prior to the breaking of the specimen. Stress corresponding to the ultimate load is known as ultimate stress.

Working stress or Safe stress

Working stress= Yield stress/Factor of safety.

5.1 Volumetric Stress

Three mutually perpendicular like direct stresses of same intensity produced in a body constitute a volumetric stress. For example consider a body in the shape of a cube subjected equal normal pushes on all its six faces. It is now subjected to equal compressive stresses p in all the three mutually perpendicular directions. The body is now said to be subjected to a volumetric compressive stress p.

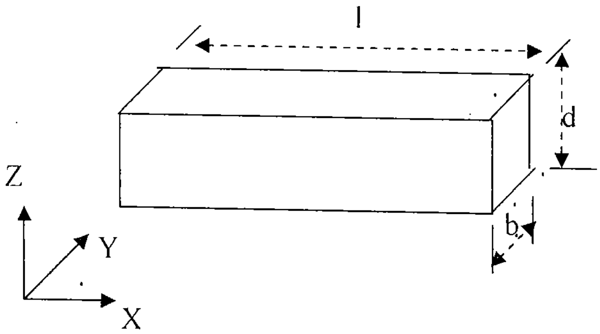


Volumetric stress produces a change in volume of the body without producing any distortion to the shape of the body.

5.2. Volumetric Strain

Volumetric strain of a deformed body is defined as the ratio of the change in volume of the body to the deformation to its original volume. If V is the original volume and δV the change in volume occurred due to the deformation, the volumetric strain e_v induced is given by $\varepsilon_V = \delta V / V$

Consider a uniform rectangular bar of length l, breadth b and depth d as shown in figure. Its volume V is given by,



$$\varepsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

$$\varepsilon_y = -\mu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

$$\varepsilon_z = -\mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1}{E}(\sigma_x + \sigma_y + \sigma_z)(1-2\mu$$

$$V = lbd$$

$$\delta V = \delta l\, bd + \delta b\, ld + \delta d\, lb$$

$$\delta V / V = (\delta l / l) + (\delta b / b) + (\delta d / d)$$

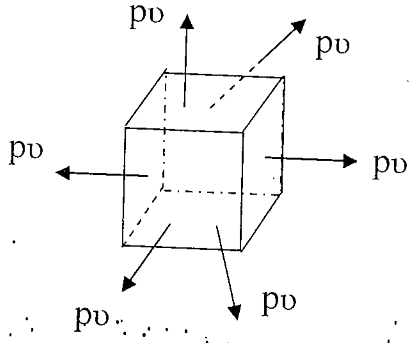
$$\varepsilon_V = \varepsilon_X + \varepsilon_y + \varepsilon_z$$

This means that volumetric strain of a deformed body is the sum of the linear strains in three mutually perpendicular directions.

5.3 Bulk Modulus (K)

Bulk Modulus is the ratio of volumetric stress to volumetric strain within the elastic limit. If p_v is the volumetric stress within elastic limit and e_v the corresponding volumetric strain, we have $K = \sigma_v / \varepsilon_v$.

5.4. Relationship between Modulus of Elasticity E and Bulk Modulus K



Consider a cube element subjected to volumetric tensile stress $\sigma\upsilon$ in X,Y and Z directions. Stress in each direction is equal to $p\upsilon$. ie $\sigma x = \sigma y = \sigma z = \sigma\upsilon$

Consider strains induced in X-direction by these stresses. σx induces tensile strain , while σy and σz induces compressive strains. Therefore,

$\varepsilon x = \sigma x/E - \mu[\sigma y/E + \sigma z/E] = \sigma\upsilon/E[1-2\mu]$

due to the perfect symmetry in geometry and stresses

$\varepsilon y = \sigma\upsilon/E[1-2\mu]$

$\varepsilon z = \sigma\upsilon/E[1-2\mu]$

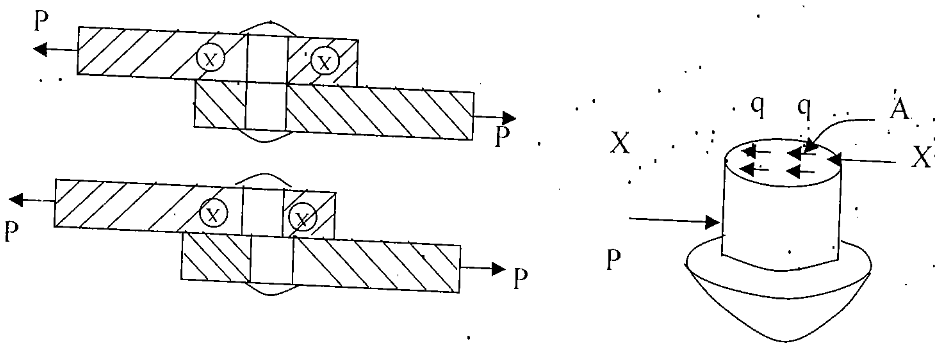
Bulk Modulus K = Volumatri stress / Volumatric strain = $\sigma\upsilon/\varepsilon\upsilon$

$K = \sigma v/(\varepsilon_x + \varepsilon_y + \varepsilon_z) = \sigma\upsilon/[3\sigma\upsilon/E(1-2\mu)]$

ie $E = 3K(1-2\mu)$ is the required relationship.

5.5 Shear Stress

The internal resistance acting tangential to the cross section is called shear stress. Consider the section x-x of the rivet forming joint between two plates subjected to a tensile force P as shown in figure.

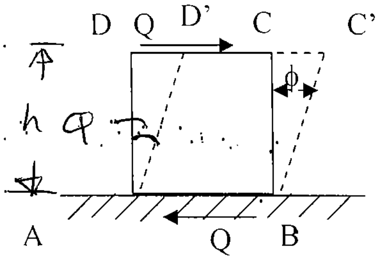


The stresses set up at the section x-x acts along the surface of the section, that is, along a direction tangential to the section. It is specifically called shear or tangential stress at the section and is denoted by q.

Shear stress $\tau = R/A = P/A$

5.6Shear Strain

Shear strain is defined as the strain accompanying a shearing action. It is the angle in radian measure through which the body gets distorted when subjected to an external shearing action. It is denoted by Φ,



$CC' = DD' = \delta$

$\tan\varphi = \frac{CC'}{BC} \cong \varphi =$

Consider a cube ABCD subjected to equal and opposite forces Q across the top and bottom forces AB and CD. If the bottom face is taken fixed, the cube gets distorted through angle ϕ to the shape ABC'D'. Now strain or deformation per unit length is
Shear strain of cube = CC' / CD = CC' / BC = ϕ radian

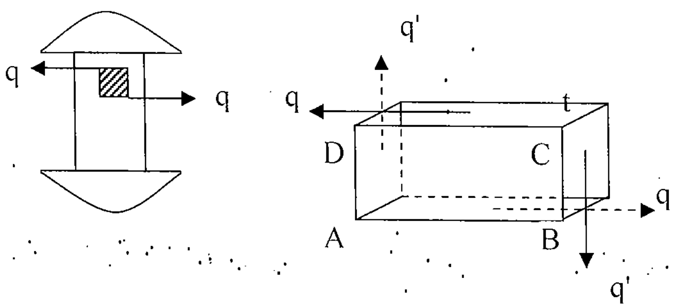
5.7 Modulus of Rigidity or Shear Modulus (G)

Modulus of Rigidity is the ratio of shear stress to shear strain within elastic limit. It is denoted by N,C or G. if τ is the shear stress within elastic limit and ϕ the corresponding shear strain; then **G = τ/ϕ**.

Modulus of rigidity or Shear modulus may be defined as the shear stress required to produce one radian of shear strain.

6. Complementary shear stress

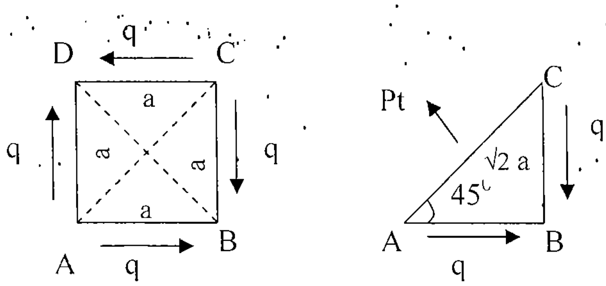
In a state of simple shear , a shear stress (q) of any intensity along a plane is always accompanied by a complementary shear stress (q') of same intensity along a plane at right angles to the plane. Ie .q =q'



6.1 Direct Stresses Developed Due to Simple Shear.

Consider a square element of side a and thickness t in a state of simple shear as shown in figure. It is clear that the shear stress on the forces of element causes it to elongate in the direction of the diagonal BD. Therefore a tensile stress of same intensity pt is induced in the elements along BD. ie, across the plane of the diagonal AC.

The triangular portion ABC of the element is in equilibrium under the action of the following.



pt = q

It can also be seen that the shear stress on the faces of the element causes it to foreshorten in the direction of the diagonal BD. Therefore a compressive stress pc is induced in the element in the direction AC, ie across the plane of the diagonal BD. It can also be shown that pc = q.

It can thus be concluded that simple shear of any intensity gives rise to direct stresses of same intensity along the two planes inclined at 45° to the shearing plane. The stress along one of these planes being tensile and that along the other being compressive.

7. Elastic Constants

Elastic constants are used to express the relationship between stresses and strains. Young's modulus E, Shear moudulus G and Bulk modulus K which we may call the elastic modulae corresponding to three distinct types of stresses and strains.

The elastic constants are E,G,K and poison's ratio(μ).

7.1 Relationship among the elastic constants

The relation between modulus of elasticity (E) and modulus of rigidity(G) is given by

$$E = 2G(1 + \mu) \quad \text{or} \quad G = E/2(1 + \mu)$$

The relation between modulus of elasticity (E) and Bulk modulus (K) is given by

$$E = 3K(1 - 2\mu)$$

The relation between modulus of elasticity (E), modulus of rigidity(G) and Bulk modulus (K) is given by $E = 9KG/(3K+G)$

S.M. - 1 UNIT - 1 ASSIGNMENT

1. A 2m long bar of uniform section 50 mm2 extends 2mm under a limiting axial stress of 200N/mm2. What is the modulus of resilience for the bar..
2. Derive the expression for temperature stresses of a bar.
3. A material has modulus of rigidity equal to 0.4 x 105 N/mm2 and bulk modulus equal to 0.75 x 105 N/mm2. Find the Young's Modulus and Poisson's Ratio.
4. A steel rod 30 mm diameter and 300mm long is subjected to tensile force P acting axially. The temperature of the rod is then raised through 600C and total extension measured as 0.30mm. Calculate the value of tensile force P. Take ES for of steel =200 GN/m2 and thermal coefficient of steel $=12 \times 10^{-6}$ /°C
5. A hollow cast-iron cylinder 4 m long, 300 mm outer diameter, and thickness of metal 50 mm is subjected to a central load on the top when standing straight. The stress produced is 75000 kN/m2. Assume Young's modulus of cast iron as 1.5 x 108 KN/m2, find
i) Magnitude of the load, ii) Longitudinal strain produced and iii) Total decrease in length
6. A circul rod 0.2m long, tapers form 20mm diameter at one end to 10mm at other end. On applying an axial pull of 6KN it was found to extend by 0.068mm. Determine the Young's Modulus of the rod material.