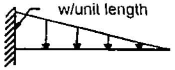
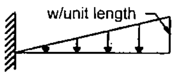
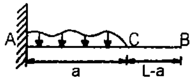
SA-I formulas



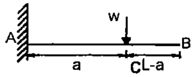
$$\theta_B = \frac{wL^3}{24EI} \qquad y_B = \frac{wL^4}{30EI}$$



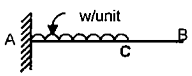
$$\theta_B = \frac{wL^3}{6EI} - \frac{wL^3}{24EI} = \frac{wL^3}{8EI}$$
$$y_B = \frac{wL^4}{8EI} - \frac{wL^4}{30EI} = \frac{11wL^4}{120EI}$$



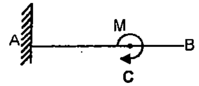
$$\theta_C = \theta_B \qquad y_B = y_B + \theta_C (L-a)$$



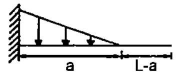
$$\theta_C = \theta_B = \frac{Wa^2}{2EI} \qquad y_C = \frac{Wa^3}{3EI}$$
$$y_B = y_C + \theta_C (L-a)$$
$$= \frac{Wa^3}{3EI} + \frac{Wa^2}{2EI}(L-a) = \frac{Wa^2}{2EI}(L - \frac{a}{3})$$



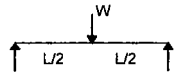
$$\theta_C = \theta_B = \frac{wa^3}{6EI} \qquad y_C = \frac{wa^4}{8EI}$$
$$y_B = \frac{wa^4}{8EI} + \frac{wa^3}{6EI}(L-a) = \frac{wa^3}{6EI}(L - \frac{a}{4})$$



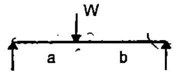
$$\theta_c = \theta_B = \frac{Ma}{EI} \qquad y_c = \frac{Ma^2}{2EI}$$
$$y_B = \frac{Ma^2}{2EI} + \frac{Ma}{EI}(L-a) = \frac{Ma}{EI}\left(L - \frac{a}{2}\right)$$



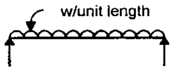
$$\theta_B = \theta_C = \frac{wa^3}{24EI} \qquad y_C = \frac{wa^4}{30EI}$$
$$y_B = y_C + \theta_C (L-a) = \frac{wa^4}{30EI} + \frac{wa^3}{24EI}(L-a)$$
$$= \frac{wa^3}{24EI}(L - \frac{a}{5})$$



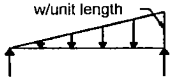
$$\theta_A = \theta_B = \frac{WL^2}{16EI} \qquad y_C = \frac{WL^3}{48EI}$$



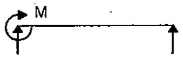
$$\theta_A = \frac{Wab(L+b)}{6L\,EI} \qquad \theta_B = \frac{Wab(L+a)}{6L\,EI}$$
$$\theta_C = \frac{Wab(b-a)}{3L\,EI} \qquad y_C = \frac{Wa^2b^2}{3L\,EI}$$



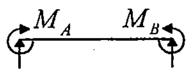
$$\theta_A = \frac{wL^3}{24EI} \qquad y_c = y_{max} = \frac{5wL^4}{384EI}$$
$$\theta_B = \frac{wL^3}{24EI}$$



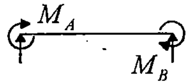
$$\theta_A = \frac{7wL^3}{360EI} \qquad \theta_B = \frac{8wL^3}{360EI}$$



$$\theta_A = \frac{ML}{3EI} \qquad \theta_B = \frac{ML}{6EI}$$

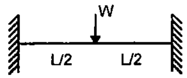
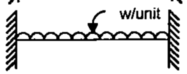
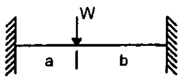
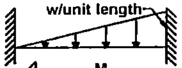
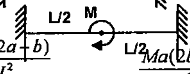


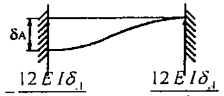
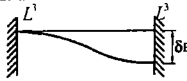
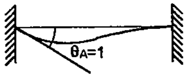
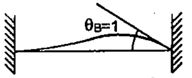
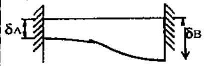
$$\theta_A = \frac{M_A L}{3EI} + \frac{M_B L}{6EI}$$
$$\theta_B = \frac{M_A L}{6EI} + \frac{M_B L}{3EI}$$



$$\theta_A = \frac{M_A L}{3EI} - \frac{M_B L}{6EI}$$
$$\theta_B = \frac{M_A L}{6EI} - \frac{M_B L}{3EI}$$

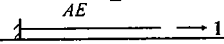
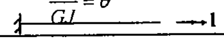
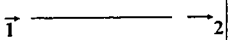
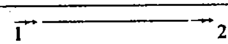
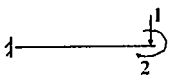
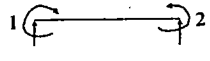
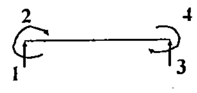
FIXED END MOMENTS

STRUCTURE	Sagging BM M_A	Sagging BM M_B	Clockwise moment M_{FAB}	Clockwise moment M_{FBA}
	$-\frac{WL}{8}$	$-\frac{WL}{8}$	$-\frac{WL}{8}$	$\frac{WL}{8}$
	$-\frac{wL^2}{12}$	$-\frac{wL^2}{12}$	$-\frac{wL^2}{12}$	$\frac{wL^2}{12}$
	$-\frac{Wab^2}{L^2}$	$-\frac{Wa^2b}{L^2}$	$-\frac{Wab^2}{L^2}$	$\frac{Wa^2b}{L^2}$
	$-\frac{wL^2}{30}$	$-\frac{wL^2}{20}$	$-\frac{wL^2}{30}$	$\frac{wL^2}{20}$
 $\frac{Mb(2a-b)}{L^2}$ $\frac{Ma(2b-a)}{L^2}$	$\frac{M}{4}$	$-\frac{M}{4}$	$\frac{M}{4}$	$\frac{M}{4}$

STRUCTURE	Sagging BM M_A	Sagging BM M_B	Clockwise moment M_{FAB}	Clockwise moment M_{FBA}
 $-\frac{12EI\delta_A}{L^3}$ $\frac{12EI\delta_A}{L^3}$	$\frac{6EI\delta_A}{L^2}$	$-\frac{6EI\delta_A}{L^2}$	$\frac{6EI\delta_A}{L^2}$	$\frac{6EI\delta_A}{L^2}$
	$-\frac{6EI\delta_B}{L^2}$	$\frac{6EI\delta_B}{L^2}$	$-\frac{6EI\delta_B}{L^2}$	$-\frac{6EI\delta_B}{L^2}$
	$\frac{4EI\theta_A}{L}$	$-\frac{2EI\theta_A}{L}$	$\frac{4EI\theta_A}{L}$	$\frac{2EI\theta_A}{L}$
	$\frac{2EI\theta_B}{L}$	$-\frac{4EI\theta_B}{L}$	$\frac{2EI\theta_B}{L}$	$\frac{4EI\theta_B}{L}$
	$\frac{6EI(\delta_A-\delta_B)}{L^2}$	$-\frac{6EI(\delta_A-\delta_B)}{L^2}$	$\frac{6EI(\delta_A-\delta_B)}{L^2}$	$\frac{6EI(\delta_A-\delta_B)}{L^2}$

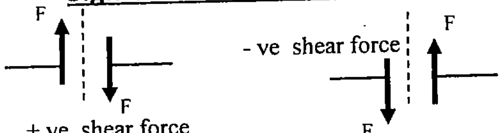
Flexibility influence coefficient F_{ij} is defined as displacement produced at i due to unit force at j.

Stiffness influence coefficient K_{ij} is defined as force produced at i due to unit displacement at j.

Sl.No.	Structure	Flexibility Matrix	Stiffness Matrix
01.	$\frac{PL}{AE} = \Delta$ 	$[F_m] = \left[\frac{L}{AE}\right]$	$[K_m] = \left[\frac{AE}{L}\right]$
02.	$\frac{TL}{GJ} = \theta$ 	$[F_m] = \left[\frac{L}{GJ}\right]$	$[K_m] = \left[\frac{GJ}{L}\right]$
03.		—	$[K_m] = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$
04.			$[K_m] = \frac{GJ}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$
05.		$[F_m] = \begin{bmatrix} \frac{L^3}{3EI} & \frac{L^2}{2EI} \\ \frac{L^2}{2EI} & \frac{L}{EI} \end{bmatrix}$	$[K_m] = \begin{bmatrix} \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$
06.		$[F_m] = \begin{bmatrix} \frac{L}{AE} & 0 & 0 \\ 0 & \frac{L^3}{3EI} & \frac{L^2}{2EI} \\ 0 & \frac{L^2}{2EI} & \frac{L}{EI} \end{bmatrix}$	$[K_m] = \begin{bmatrix} \frac{AE}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$
07.		$[F_m] = \begin{bmatrix} \frac{L}{3EI} & -\frac{L}{6EI} \\ -\frac{L}{6EI} & \frac{L}{3EI} \end{bmatrix}$	$[K_m] = \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} \\ \frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix}$
08.		$[F_m] = \begin{bmatrix} \frac{L}{3EI} & \frac{L}{6EI} \\ \frac{L}{6EI} & \frac{L}{3EI} \end{bmatrix}$	$[K_m] = \begin{bmatrix} \frac{4EI}{L} & -\frac{2EI}{L} \\ -\frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix}$
09.		$[k_m] = \begin{bmatrix} \frac{12EI}{L^3} & -\frac{6EI}{L^2} & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ -\frac{6EI}{L^2} & \frac{4EI}{L} & \frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ -\frac{6EI}{L^2} & \frac{2EI}{L} & \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$	

Shear force at a section: The algebraic sum of the vertical forces acting on the beam either to the left or right of the section is known as the *shear force at a section.*

Sign Convention for shear force



Bending moment (BM) at section: The algebraic sum of the moments of all forces acting on the beam either to the left or right of the section is known as the *bending moment at a section*

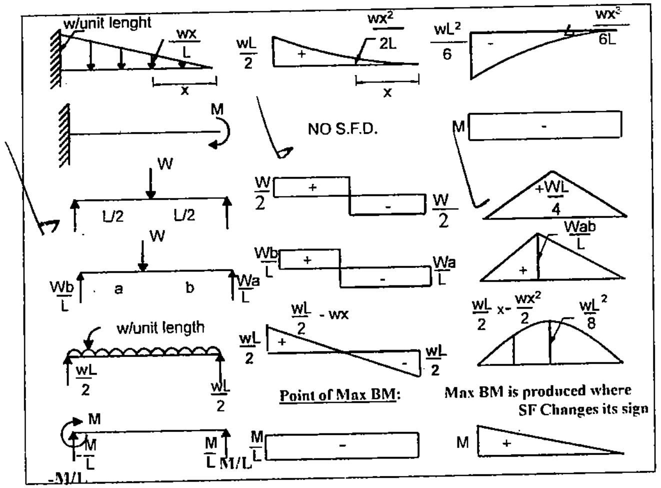
Bending Moment (BM): The moment which causes the bending effect on the beam is called *Bending Moment.* It is generally denoted by 'M' or 'BM'.

Sagging bending moment [Positive bending moment]

Hogging bending moment [Negative bending moment]

Intensity of Load	0	Constant	Linear
Load			
Shear	Constant	Linear	Parabolic
Moment	Linear	Parabolic	Cubic

Intensity of Load	0	0	Constant	Linear
Load	M			
Shear	Constant	Constant	Linear	Parabolic(2)
Moment	Linear	Linear	Parabolic	Cubic(3)



Internal forces at a section: Axial force, *Shear force and Bending Moment are internal forces at a section of the beam.* The following is the +ve sign convention

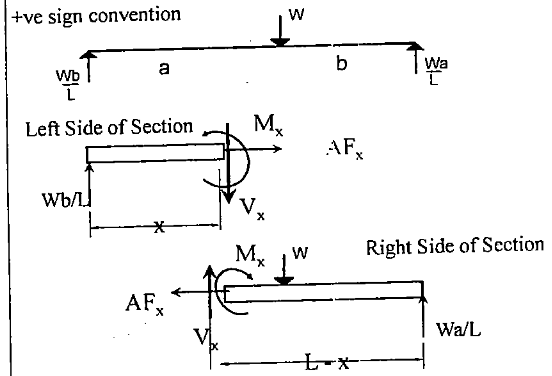


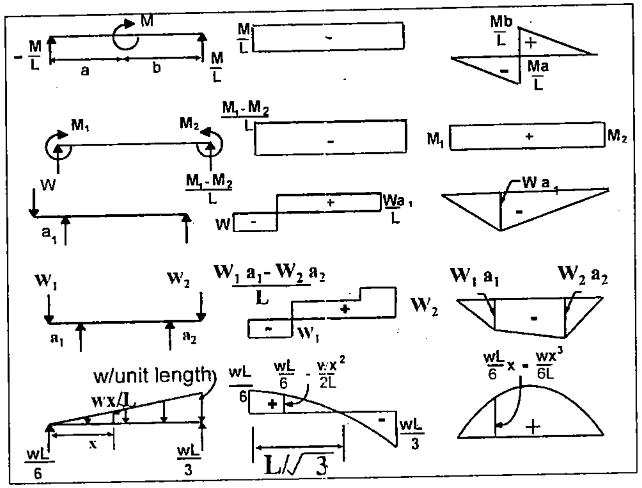
Table: Variation of Shear force and bending moments

Type of load / SFD/BMD	Between point loads OR for no load region	Uniformly distributed load	Uniformly varying load
Shear Force Diagram	Horizontal line	Inclined line	Two-degree curve (Parabola)
Bending Moment Diagram	Inclined line	Two-degree curve (Parabola)	Three-degree curve (Cubic-parabola)

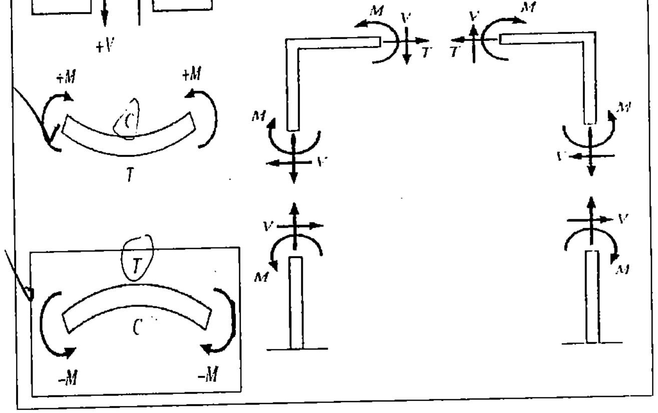
Point of Contra flexure [Inflection point]:

It is the point on the bending moment diagram where bending moment changes the sign from positive to negative or vice versa.

It is also called 'Inflection point'. At the point of inflection point or contra flexure the bending moment changes its sign and magnitude is zero.



Sign Convention for Shear Force and Bending Moment



Sections for Shear Force and Bending Moment Calculations:

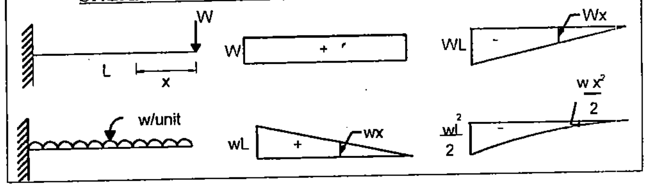
Shear force and bending moments are to be calculated at various sections of the beam to draw shear force and bending moment diagrams.

These sections are generally considered on the beam where the magnitude of shear force and bending moments are changing abruptly.

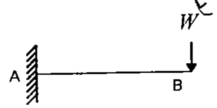
Therefore these sections for the calculation of shear forces include sections on either side of point load, uniformly distributed load or uniformly varying load where the magnitude of shear force changes abruptly.

The sections for the calculation of bending moment include position of point loads, either side of uniformly distributed load, uniformly varying load and couple

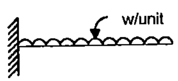
SHEAR FORCE & BENDING MOMENT DIAGRAMS



DEFLECTION OF BEAMS



$$\theta_B = \frac{WL^2}{2EI} \qquad y_B = \frac{WL^3}{3EI}$$



$$\theta_B = \frac{wL^3}{6EI} \qquad y_B = \frac{wL^4}{8EI}$$

$$\theta_B = \frac{ML}{EI} \qquad y_B = \frac{ML^2}{2EI}$$