2. Submerged backfill

If the water table exists at depth H_1 below the surface of backfill, then we have

For $0 \leq z \leq H_1$

$p_a = \gamma z \tan^2\alpha + 2c\tan\alpha$

For $H_1 \leq z \leq H$

$$p_a = [\gamma H_1 \tan^2\alpha + \gamma^1(z-H_1)\tan^2\alpha + q\tan^2\alpha + \gamma_w(z-H_1) + 2c\tan\alpha]$$

4.4 Coulomb's Wedge Theory:

Rankine (1860) in his theory of earth pressure considered the stresses acting on an element and their relationship in the plastic equilibrium state. Earlier to this Coulomb (1776) proposed the wedge theory in which he assumed that a portion of soil mass adjacent to the retaining wall breaks away from the rest of the soil mass. By considering the forces acting on this soil wedge in the limiting equilibrium condition the lateral earth pressure is computed.

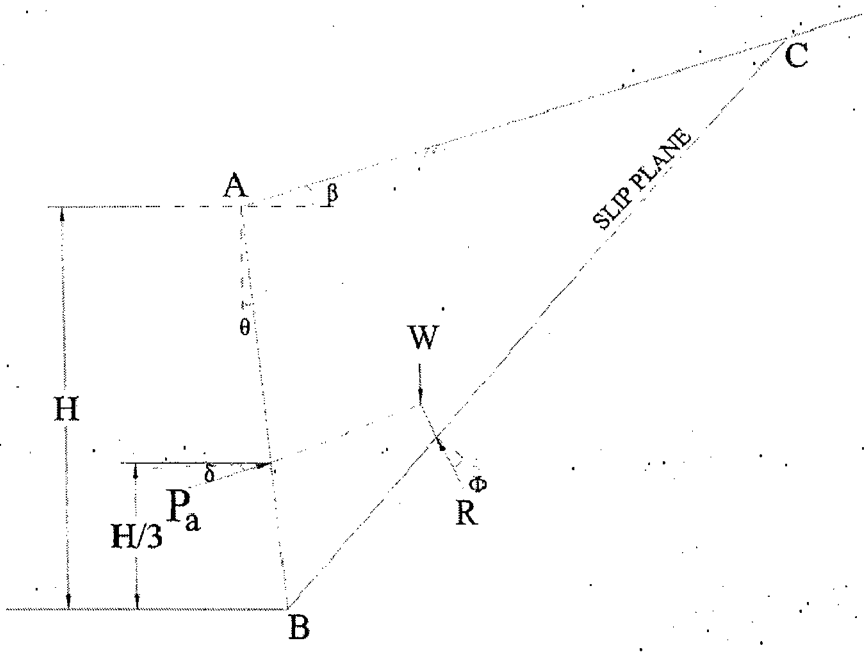

Fig 4.19 Free body diagram of sliding wedge

In Fig 4.19 ABC is the sliding wedge. Coulomb assumed that active earth pressure is caused when the wall tends to move downward and outward. On the other hand passive earth pressure is caused when the wall moves upward and inward. Fig 4.19 is the free body diagram of the sliding wedge in the limiting equilibrium condition for the active state.

Assumptions made in Coulomb's theory:

1. The backfill is cohesionless, dry, homogenous, isotropic and elastically undeformable but breakable.
2. The slip surface is a plane which passes through the heel of wall.
3. The sliding wedge behaves like a rigid body and the earth pressure can be computed by considering the limiting equilibrium of the wedge as a whole.
4. The back of the wall is rough.
5. The position and direction of the resultant earth pressure are known. It acts at distance one-third the height of the wall above base and is inclined at an angle δ to the normal to the back of wall, where δ is the angle of wall friction.
6. In the limiting equilibrium condition the sliding wedge is acted upon by three forces as shown in Fig 4.19.
    - (i) Weight W of the sliding wedge acting vertically through its centre of gravity.
    - (ii) The resultant active earth pressure P_a acting at distance $\frac{H}{3}$ above base and inclined at an angle δ to the normal to the back of wall.
    - (iii) The resultant reaction R inclined at an angle $\emptyset$ to the normal to the slip plane and passing through the point of intersection of the other two forces.

For the condition of yield of the base of wall and wall movement away from fill, the most dangerous or the critical slip plane is that for which the wall reaction is maximum. The active earth pressure is computed as the maximum lateral pressure which the wall must resist before it moves away from the fill.

4.5 Condition for Maximum Pressure from Sliding Wedge

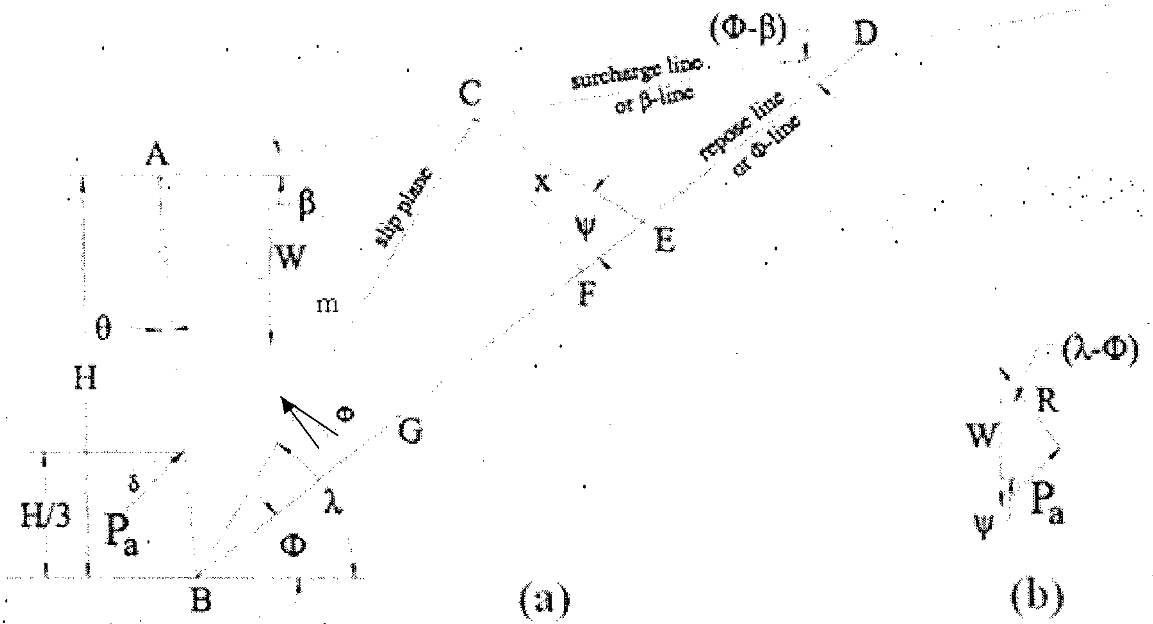

Fig 4.20 Condition for maximum pressure from sliding wedge - P_a

In Fig 4.20(a) AB is the back of the wall with positive batter angle θ. AD is surface of backfill inclined at an angle β to horizontal, and referred to as surcharge line. BD is inclined at angle Ø to the horizontal and is called repose line as it is the slope with which soil rests without any lateral support. Let BC be the slip plane or rupture plane inclined at angle □ to the horizontal. We have to determine the position of slip plane for which the sliding wedge exerts maximum pressure on the wall. □ is referred to as critical slip angle. It is clear from Fig 4.20(a) that the critical slip plane lies between repose line (□=Ø) and back of wall (□=90°+θ). Further we observe that P_a is inclined to the vertical at an angle (90-θ-δ) which is denoted by φ. The reaction R is inclined to the vertical at (□-Ø). The triangle of forces is shown in Fig 4.20(b). In Fig 4.20(a) CE is drawn making angle φ with the Ø- line. Let x and m be the length of perpendiculars drawn from C and A to BD. Let BD be n. The triangle BCE and triangle of forces are similar. Therefore, we have

$$\frac{P_a}{CE} = \frac{W}{BE}.$$

i.e., $$\frac{P_a}{CE} = W.\left(\frac{CE}{BE}\right)$$ Eq 4.25

From Δ^{le} CFE , $\sin\varphi = \frac{x}{CE}$

$\therefore CE = \frac{x}{\sin\varphi} = x \operatorname{cosec}\varphi$

or $CE = A_1x$ Eq 4.26

where $A_1 = \operatorname{cosec}\varphi$

BE =BD – FD + FE

From Δ^{le} CFD, $\tan(\emptyset-\beta) = \frac{x}{FD}$

$\therefore FD = x \cot(\emptyset-\beta)$

From Δ^{le} CFE, $\tan\varphi = \frac{x}{FE}$

$\therefore FE = x \cot\varphi$

Hence, $BE = n - x[\cot(\emptyset-\beta) - \cot\varphi]$

or $BE = n - A_2x$ Eq 4.27

where $A_2 = [\cot(\emptyset-\beta) - \cot\varphi]$

$W = \gamma(\Delta\ ABC) = \gamma[\Delta\ ABD - \Delta\ BCD]$

i.e $W = \frac{1}{2}\gamma(m-x)n$ Eq 4.28

Substituting Eqns 4.26, 4.27 and 4.28 in Eqn 4.25, we get

$$P_a = \frac{1}{2}\gamma(m-x)n\ \frac{A_1 x}{n-A_2 x} = \left(\frac{1}{2}\gamma n A_1\right)\left(\frac{mx-x^2}{n-A_2 x}\right)$$

In the last equation x is the only variable which depends on the position of slip plane.

For maxima $\frac{dP_a}{dx} = 0$

$$\therefore \frac{dP_a}{dx} = \left(\frac{1}{2}\gamma n A_1\right)\frac{(m-2x)(n-A_2x)-(-A_2)(mx-x^2)}{(n-A_2x)^2} = 0$$

$\therefore (m - 2x)(n - A_2 x) = -A_2(mx - x^2)$

mn - A_2mx – 2nx + $2A_2x^2$ = -A_2 mx + A_2x^2

mn – 2xn = - A_2x^2 .

Rearranging,

mn – xn = xn - A_2x^2 = x(n - A_2x) = $x \times BE$

We can write

$\frac{mn}{2} - \frac{xn}{2} = x\frac{BE}{2}$

or Δ ABD - Δ BCD = Δ BCE

i.e., Δ ABC = Δ BCE Eq 4.29

Thus the condition for the sliding wedge ABC to exert maximum pressure (P_a) on wall is that the slip plane BC is located such that triangles ABC and BCE are equal in area. Rebhann (1871) is credited to have presented this proof.

4.6 Rebhann's Graphical Method for Active Earth Pressure of Cohesionless Soil

Rebhann (1871) gave this graphical procedure for locating the slip plane and determining the total active earth pressure according to Coulomb's wedge theory.

Referring to Fig 4.21 the steps involved in the graphical procedure is

1. Given the height H and batter angle θ the back AB of the wall is constructed.
2. Through A , surcharge line or β-line is drawn inclined at an angle β to the horizontal.
3. Through B, repose line or Ø-line is drawn inclined at an angle Ø to the horizontal, intersecting the β-line at D.

$\frac{H}{AB} = \cos\theta$

$\therefore \quad AB = \frac{H}{\cos\theta}$

$\frac{H_1}{AB} = \cos(\beta - \theta)$

$\therefore \quad H_1 = AB\cos(\beta - \theta) = \frac{H\cos(\beta-\theta)}{\cos\theta}$

The effect of surcharge load is taken into account by replacing γ by γ_e while computing P_a.

Note: Ifthe surcharge load extends beyond C, the value of L in (qL) should be taken equal to AC.

4.7 Culmann's Graphical Method for Active Earth Pressure of Cohesionless Soil Based on Coulomb's Wedge Theory

This graphical method given by Culmann (1886) is more general than Rebhann's method and is very convenient to use in the case of layered backfill, backfill with breaks at surface and different types of surcharge load.

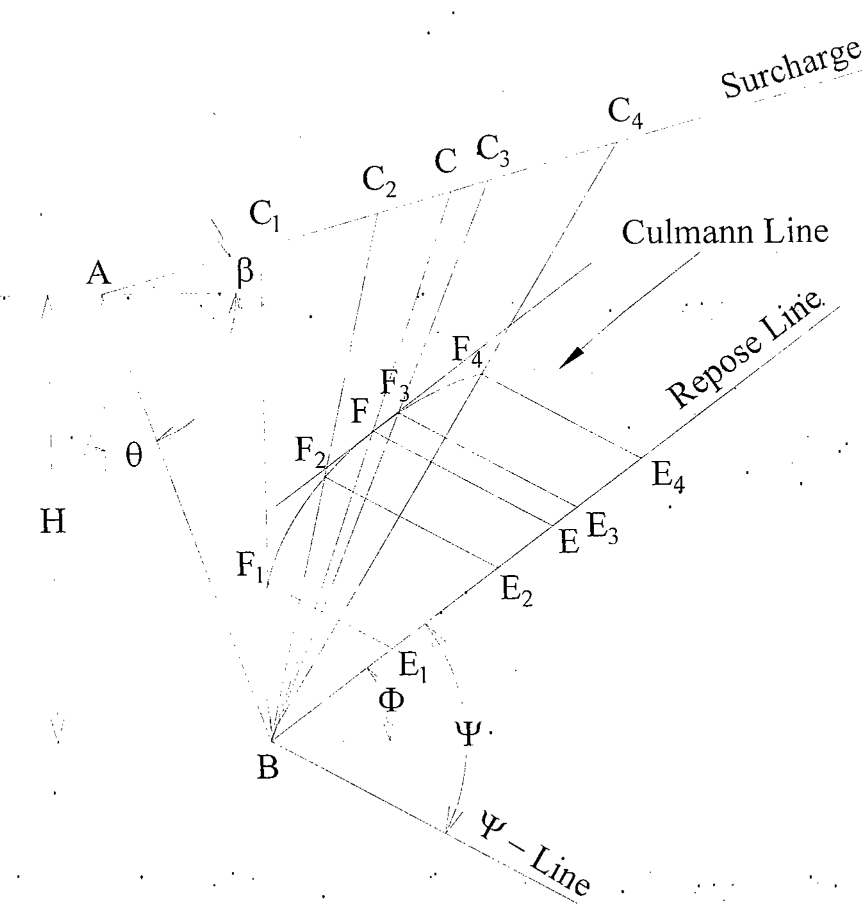

Fig 4.26 Culmann's graphical method

The steps involved in the Culmann's method are as follows:

1. Given height H and batter angle θ , the back AB of the wall is constructed.
2. Through A, the surcharge line (β-line) is drawn inclined at angle β to the horizontal.
3. Through B, the repose line (∅-line) is drawn inclined at an angle ∅ to the horizontal.
4. Again through B, the ψ-line is drawn inclined at an angle ψ to the ∅-line ($\psi = 90° - \theta - \delta$).
5. Trial slip planes BC_1, BC_2...... are drawn. The weights of the wedges ABC_1, ABC_2...... are calculated and plotted to scale as BE_1, BE_2,...... on the ∅-line.
6. Through E_1, E_2,.... lines are drawn parallel to ψ-line, intersecting BC_1, BC_2,....... at F_1, F_2..... respectively.
7. A smooth curveis drawn through points B, F_1, F_2...... This curve is called Culmann line.

8. A line is drawn parallel to Ø-line and tangential to Culmann line. Let it touch Culmann line at F. BF is joined and produced to intersect the β-line at C. Then BC is the critical slip plane.

9. Through F, line FE is drawn parallel to ψ-line, intersecting Ø-line at E.

10. The weight W of the wedge ABC is calculated. The resultant active earth pressure P_a is given by

$$\frac{P_a}{W} = \frac{FE}{BE}$$

$$\therefore\ P_a = W.\left(\frac{FE}{BE}\right) \qquad \text{Eq 4.39}$$

Special cases:

(i) Backfill with uniform surcharge load.

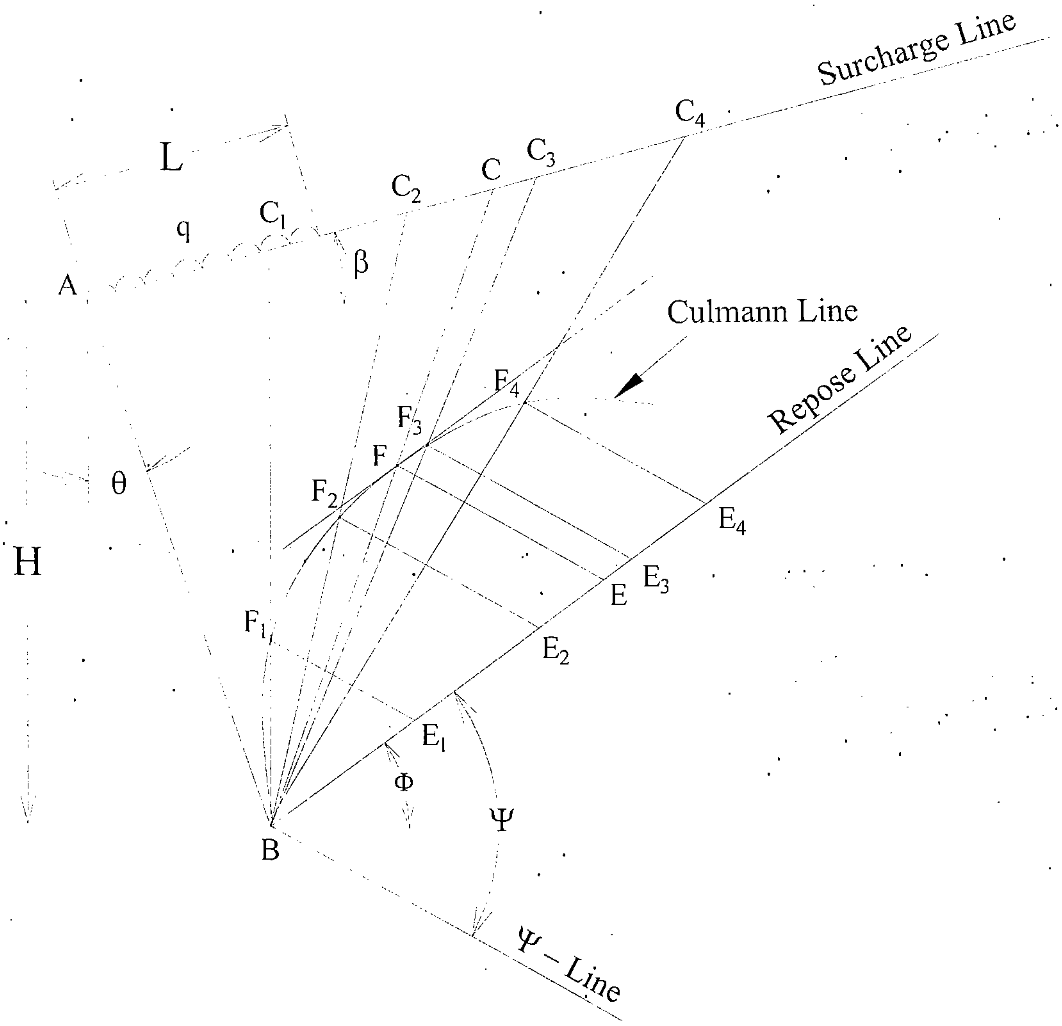

Fig 4.27 Culmann's method – Backfill with uniform surcharge load

As an illustration consider Fig 4.27 in which uniformly distributed surcharge load of intensity q is shown acting over a length L. The procedure is similar to the previous case but for the following changes.

i. BE_1 represent the sum of weight of wedge ABC_1 and surcharge load $q(AC_1)$.

ii. BE_2 represents the sum of weight of wedge ABC_2 and surcharge load qL. Similarly BE_3, BE_4 represent the sum of respective sliding wedges and surcharge load Lq.

iii. The resultant active earth pressure is given by

$$\frac{P_a}{W} = \frac{FE}{BE}$$

$$\therefore\ P_a = W.\left(\frac{FE}{BE}\right)$$

where W = (weight of wedge ABC) + (qL).

ii) Backfill with line load.

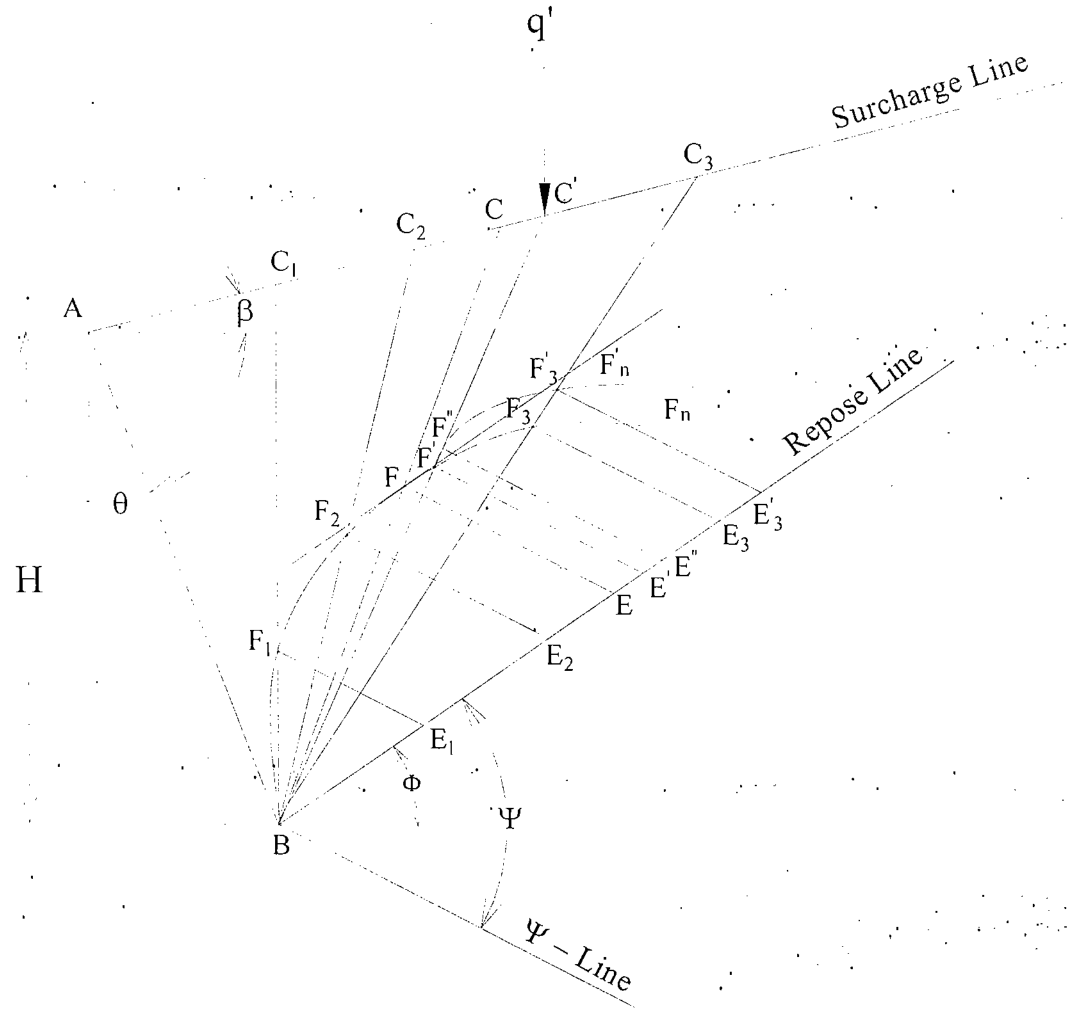

Fig 4.28 Culmann's method – effect of line load

As an illustration, consider Fig 4.28 in which a line load of intensity q^1 (per unit run) acts at distance d from top of wall. Example of line load is load due to any wall or a railway track running parallel to retaining wall. In the Fig 4.27, B, F_1, F_2…… F_n is Culmann line obtained without considering line load. BC then represents the critical slip plane and the resultant activepressure is given by

$$\therefore P_a = W \cdot \left(\frac{FE}{BE}\right) \qquad \text{Eq 4.40}$$

where W = weight of wedge ABC.

If we consider the line load then E′, E_3….. will get shifted to E″, E_3'… with E′E″ = E_3E_3' = ….. q. There is an abrupt change in the Culmann line from F_1 to F′ and $BF_1F_2F'F''.....F_n'$

represents Culmann line obtained considering the line load. If E″F″ is greater than EF, slip occurs along BC′ and the resultant active earth pressure is given by

$$P_a = W'.\left(\frac{F''E''}{BE''}\right) \qquad \text{Eq 4.41}$$

where W' = (weight of wedge ABC′) + q. On the other hand if E″F″ is less than EF, slip occurs along BC and P_a is given by Eqn 4.40.

Culmann's method can also be used to find the minimum safe distance from top of retaining wall at which the line load can be placed without causing increase in P_a.

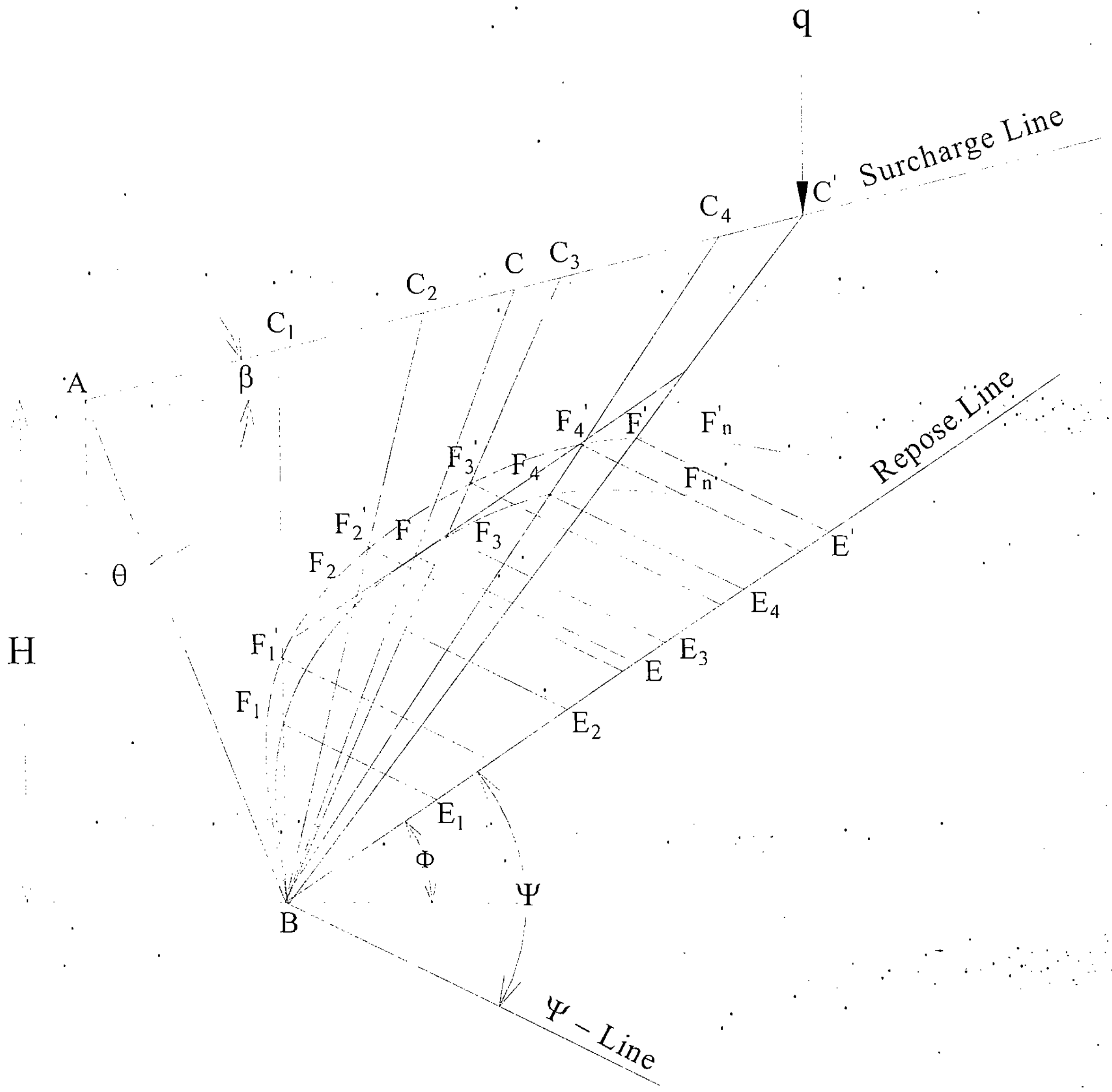

Fig 4.28a Safe location of line load

In Fig 4.28a, $BF_1F_2......F_n$ represent Culmann line obtained without considering line load. $BF_1^1F_2^1.....F_n^1$ represents Culmann line obtained by placing line load q successively at C_1, C_2........ The line drawn tangential to Culmann line $BF_1F_2......F_n$ and parallel to Ø-line touches the Culmann line at F and BC represents the critical slip plane when line load is not considered. In that case the resultant active earth pressure is given by

$$P_a = W.\left(\frac{FE}{BE}\right) \qquad \text{Eq 4.42}$$

where W= weight of wedge ABC. The tangent drawn as described above is produced to cut the Culmann line $BF_1^1F_2^1.....F_n^1$ at F^1. BF^1 is joined and produced to intersect ground line at C^1. Then AC^1 represent the minimum safe distance at which q can be located without causing increase in P_a given by Eqn 4.42

4.8 Design of Gravity Retaining Wall

Gravity retaining walls are constructed of mass concrete, brick masonry or stone masonry. A gravity retaining wall resists the lateral earth pressure by virtue of its weight. Hence it is thicker in section compared to a cantilever or counterfort R.C.Cretaining wall which resists the lateral earth pressure by virtue of its resistance to bending.

The criteria of design of gravity retaining walls are:

1. The base width of the soil must be such that the maximum pressure exerted at base on soil does not exceed the safe bearing capacity of soil.
2. No tension should develop anywhere in the base.
3. The wall must be safe against sliding.
4. The wall must be safe against overturning.

Analysis: