UNIT- 6

Boundary Layers

Learning objectives :-

At the end of this topic, you will be able to :

→ Explain the separation of boundary layer

→ Describe the effects of boundary layer separation

→ Brief about the methods of boundary layer separation

→ Describe flow in submerged bodies – drag and lift

→ understand the types of drag.

→ Describe magnus effect.

Learning Outcomes :-

By the end of this topic, you will be able to :

→ understand the separation of boundary layer.

→ know the effects of boundary layer separation.

→ List out the methods of boundary layer separation

→ understand the flow in submerged bodies – drag & lift

→ know about the types of drag

→ know magnus effect.

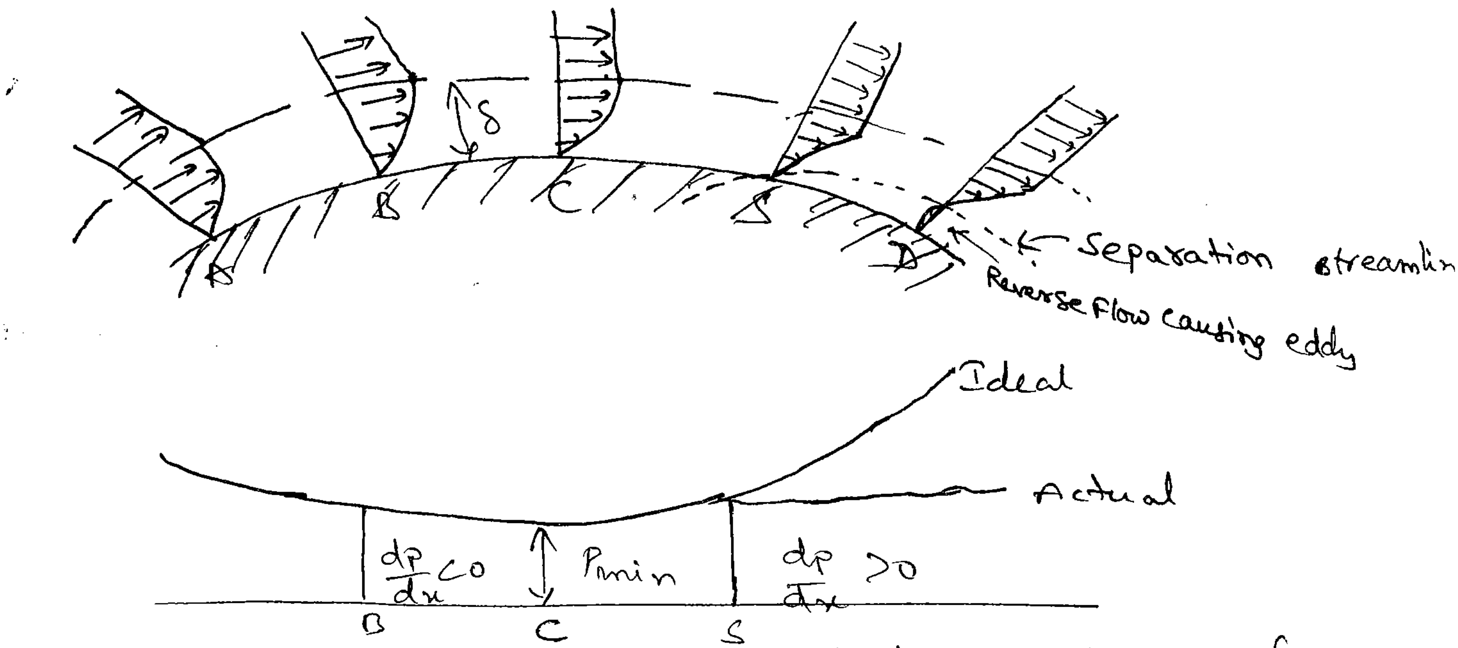
③ Separation of Boundary Layer

⇒ The boundary layer is affected by pressure gradient $\left(\frac{dp}{dx}\right)$. If the pressure gradient is zero then the boundary layer continues to grow in thickness along a flat plate.

⇒ If the pressure gradient decreases the boundary layer thickness tends to reduced.

⇒ If the pressure gradient increases the boundary layer thickness increases rapidly.

⇒ The adverse pressure gradient and the boundary shear tends bring the fluid in the boundary layer to rest. Thus the retarded fluid particles cannot penetrate into the region of high pressure owing to small kinetic energy.



⇒ Thus the boundary layer gets deflected sideways from boundary, separates from it and moves into mainstream, this phenomenon is known as separation of boundary layer.

⇒ Consider the flow of fluid over a curved surface as shown in the figure above.

⇒ The fluid flowing is accelerated upto the point where the velocity is maximum and pressure is minimum. Thus from A to C pressure gradient is negative.

⇒ However beyond the point C the net pressure in the flow increases and opposes the forward flow of liquid. Thus at a certain distance downstream of point C the fluid is completely brought to rest. The value of velocity gradient becomes zero at the point D.

⇒ Now the fluid will not be able to flow along the contour surface and separates from it.

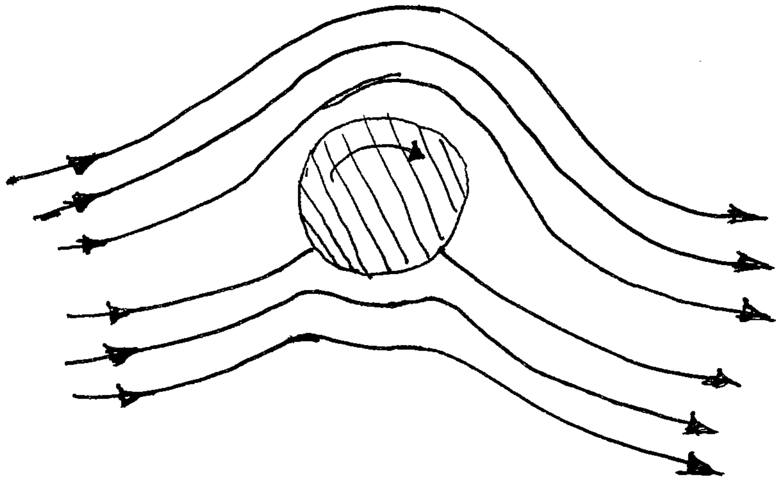
⇒ The separation first occurs at the point where the velocity gradient is zero and this point is known as separation point.

⇒ Separation occurs with both laminar & turbulent boundary layer, but the former is more susceptible to separation than latter.

⇒ The reason being the increase of velocity with the distance from boundary layer is less rapid, so the adverse pressure gradient is large, tending to separation in case of laminar boundary layer.

⇒ However incase of turbulent boundary layer velocity distribution is much more uniform because of intense lateral mixing. As a result high velocity prevails reducing the tendency of separation.

(4) (a) Ill Effects of Boundary layer Separation.



(a) Flow Past a rotating Cylinder

⇒ It gives rise to resistance to the normal flow of fluid

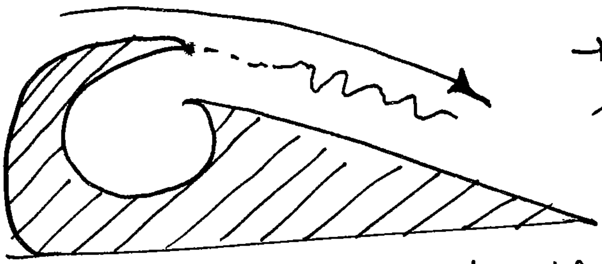
⇒ The flow pattern of fluid is affected.

→ Boundary forces are created due to pressure variation.

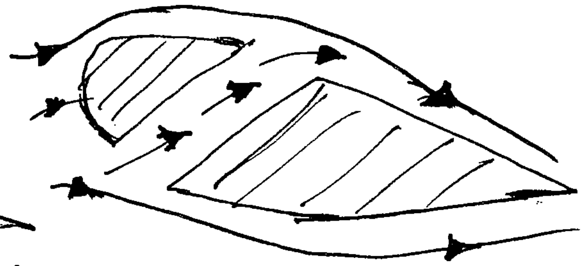
(5) Methods to control boundary layer separation.

→. Motion of solid boundary — Which involves preventing the formation of boundary layer by moving the solid boundary along with the passing fluid. As shown in fig (a).

→. Acceleration of fluid in the boundary layer - figure (B) & (c) — supplying additional energy to the fluid particles which are being retarted in the boundary layer.

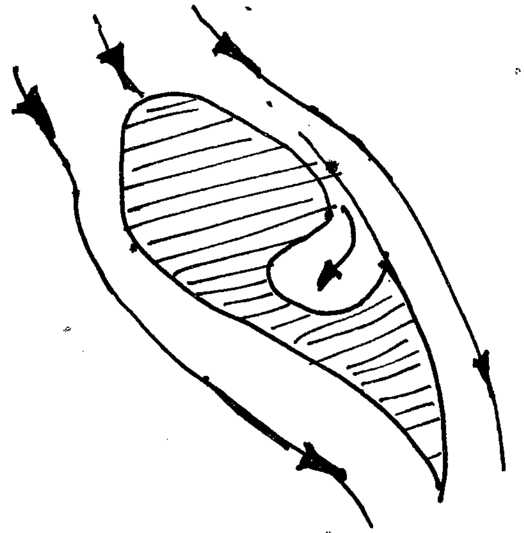


(b) Injecting fluid into boundary layer



(c) slotted wing

⇒ Suction of the fluid from the boundary layer - figure (d) - slow moving fluid from the boundary layer is remove by providing slots.

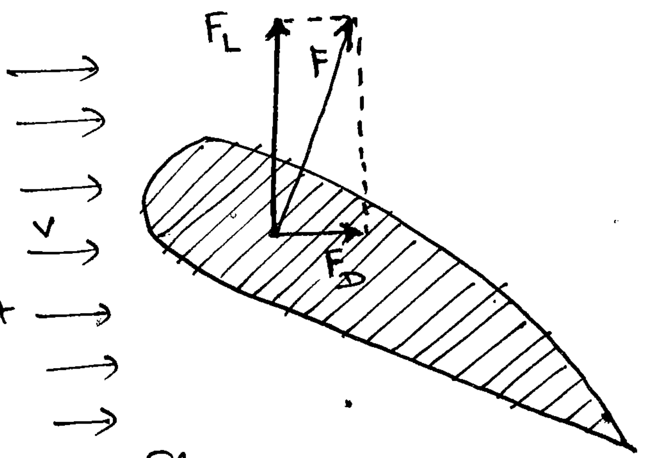


(d) suction of fluid from boundary layer.

⇒ Streamlining of bodies - By using suitably shaped bodies the point of transition from Laminar to turbulent can be reduced which results in reduced skin friction drag & streamlining prevents separation.

Flow around submerged objects - Drag and lift.

⇒ In the analysis and design of objects immersed in the fluid; the forces exerted on them by the fluid plays an significant role.



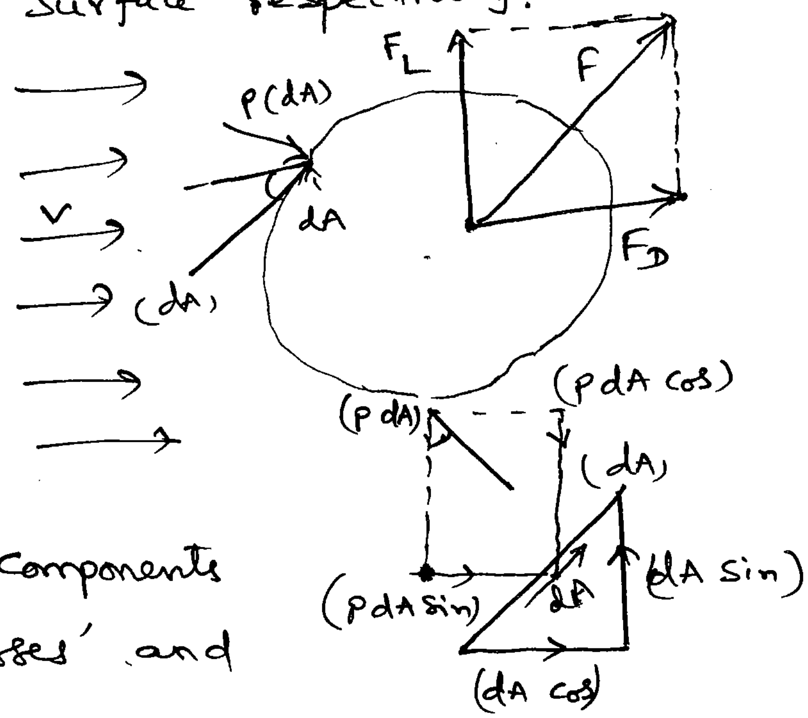
Forces on an immersed body

⇒ The force exerted by the fluid on the moving body may in general be inclined to the direction of motion & hence it has a component in the direction of motion and also perpendicular to the direction of motion as shown in fig.

⇒ The component of the force in the direction of motion is called drag F_D, and the component perpendicular to the direction of motion is called lift F_L.

⇒ However for a symmetrical body, such as sphere or cylinder, facing the flow symmetrically there is no lift and thus the total force exerted by the fluid is equal to the drag on the body.

⇒ The figure shows a body held stationary in the direction of moving fluid. The forces acting on any small element dA may be resolved into two components pdA and tdA in the normal and tangential directions to the surface respectively.



⇒ The tangential components are 'shear stresses' and the normal components are pressure forces.

⇒ The sum of the components of shear stresses in the direction of flow is called 'friction drag F_{Df}', which

4(b) For the velocity Profile given in problem 13.3 find the thickness of boundary layer at the end of the Plate and the drag force on side of a Plate 1m long and 0.8m wide when Placed in water flowing with a velocity of 150mm per Second. Calculate the value of co-efficient of drag also Take μ for water =0.01 Poises

Sol:- Given :

Length of plate $L = 1m$

width of Plate $b = 0.8m$

velocity of fluid (water) $U = 150\ mm/s = 0.15\ m/s$

μ for water $= 0.01\ \text{Poise} \Rightarrow \frac{0.01}{10}\ \frac{NS}{m^2}$

$= 0.001\ \frac{NS}{m^2}$

Reynold number at the end of the Plate i.e., at a distance of 1m from leading edge is given by

$$Re_L = \frac{\rho U L}{\mu} = \frac{1000 \times 0.15 \times 1.0}{0.001} \qquad (\because \rho = 1000)$$

$= 150000$

(i) As laminer boundary layer exists upto Reynold number $= 2\times10^5$. Hence this is the case of laminar boundary layer. Thickness of boundary layer at $x = 1.0m$ is given by equation as

$$\delta = 5.48 \frac{x}{\sqrt{Re_x}} = \frac{5.48 \times 1.0}{\sqrt{150000}}$$

$= 0.01415m$

$= 14.15mm$

(ii) Drag force on one side of the plate is given by equation

$$F_D = 0.73\, b\mu U\sqrt{\frac{\rho UL}{\mu}}$$

$$= 0.73 \times 0.8 \times 0.001 \times 0.15 \times \sqrt{150000} \qquad \left\{\because \frac{\rho UL}{\mu} = Re_L\right\}$$

$$= 0.0338$$

(iii) Co-efficient of drag, C_D is given by equation as

$$C_d = \frac{1.46}{\sqrt{Re_L}} = \frac{1.46}{\sqrt{150000}} = 0.00376$$

(1) Approximate solutions of Navier - Stokes Equation

⇒ The basic difficulty in solving Navier- stokes equations arises due to the presence of nonlinear (quadratic) inertia terms on the left hand side.

⇒ However, there are some nontrival solutions of the Navier - Stokes equations in which the nonlinear inertia terms are identically zero.

⇒ One such class of flows is termed as parallel flows in which only one velocity term is nontrivial and all the fluid particles move in one direction only.

⇒ Let us choose x to be the direction along which ~~only~~ all fluid particles travel, i.e., $u \neq 0$, $v = w = 0$.

⇒ Invoking this in continuity equation, we get,

$$\frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}}^{0} + \cancel{\frac{\partial w}{\partial z}}^{0} = 0$$

$$\therefore \frac{\partial u}{\partial x} = 0$$

⇒ Now, Navier - Stokes equations for incompressible flow becomes,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}^{0} + v\frac{\partial u}{\partial y}^{0} + w\frac{\partial u}{\partial z}^{0} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left[\frac{\partial^2 u}{\partial x^2}^{0} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right]$$

$$\frac{\partial v}{\partial t}^{0} + u\frac{\partial v}{\partial x}^{0} + v\frac{\partial v}{\partial y}^{0} + w\frac{\partial v}{\partial z}^{0} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left[\frac{\partial^2 v}{\partial x^2}^{0} + \frac{\partial^2 v}{\partial y^2}^{0} + \frac{\partial^2 v}{\partial z^2}^{0}\right]$$

$$\frac{\partial w}{\partial t}^{0} + u\frac{\partial w}{\partial x}^{0} + v\frac{\partial w}{\partial y}^{0} + w\frac{\partial w}{\partial z}^{0} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left[\frac{\partial^2 w}{\partial x^2}^{0} + \frac{\partial^2 w}{\partial y^2}^{0} + \frac{\partial^2 w}{\partial z^2}^{0}\right]$$

→ So, we obtain

$$\frac{\partial p}{\partial y} = \frac{\partial p}{\partial z} = 0 \,,$$ which means $p = p_x$ alone

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\cdot\frac{\partial p}{\partial x} + \nu\left[\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right]$$

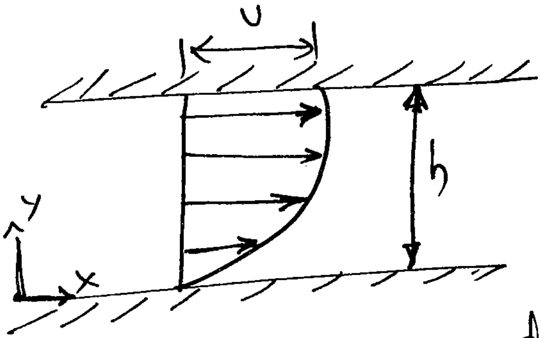
=) Equation 1 had been simplified further and an number analytical solution have been obtained. We will see two impoŕtant solutions.

<u>Coutte Flow</u>

=> A simple solution for equation 1 is obtained by couette flow between two parallel plates.

=) Here one plate is at rest and the other is moving with velocity 'U'.

=> The figure below shows coutte flow between two parallel plates



=> The velocity distribution in non-dimensional form across the channel with P as a parameter known as the non-dimensional pressure gradient, is given by,

$$\frac{u}{U} = \frac{y}{h} + P\frac{y}{h}\left(1 - \frac{y}{h}\right)$$

$$P = -\frac{h^2}{2\mu U}\left(\frac{dp}{dx}\right)$$

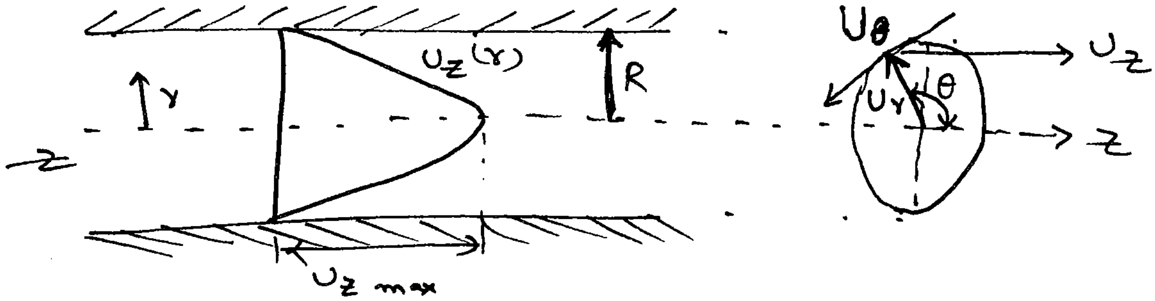
=> When $P = 0$, the velocity distribution across the channel is reduced to,

$$\frac{u}{U} = \frac{y}{h}$$

=) Equation 3 is an particular case known as Simple Coutte Flow.

⇒ Hagen Poiseuille flow :

Consider fully developed laminar flow through a straight tube of circular cross-section as in figure below.



⇒ Rotational symmetry is considered to make the flow two dimensional axisymmetric.

⇒ Let us take z-axis as the axis of the tube along which all the fluid particles travel, i.e. $V_z \neq 0$, $V_r = 0$, $V_\theta = 0$.

⇒ Now from continuity equation we obtain,

$$\frac{\partial V_r^{\,0}}{\partial r} + \frac{V_r^{\,0}}{r} + \frac{\partial V_z}{\partial z} = 0 \quad \left[\text{for rotational symmetry } \frac{1}{r} \cdot \frac{\partial V_\theta}{\partial \theta} = 0 \right]$$

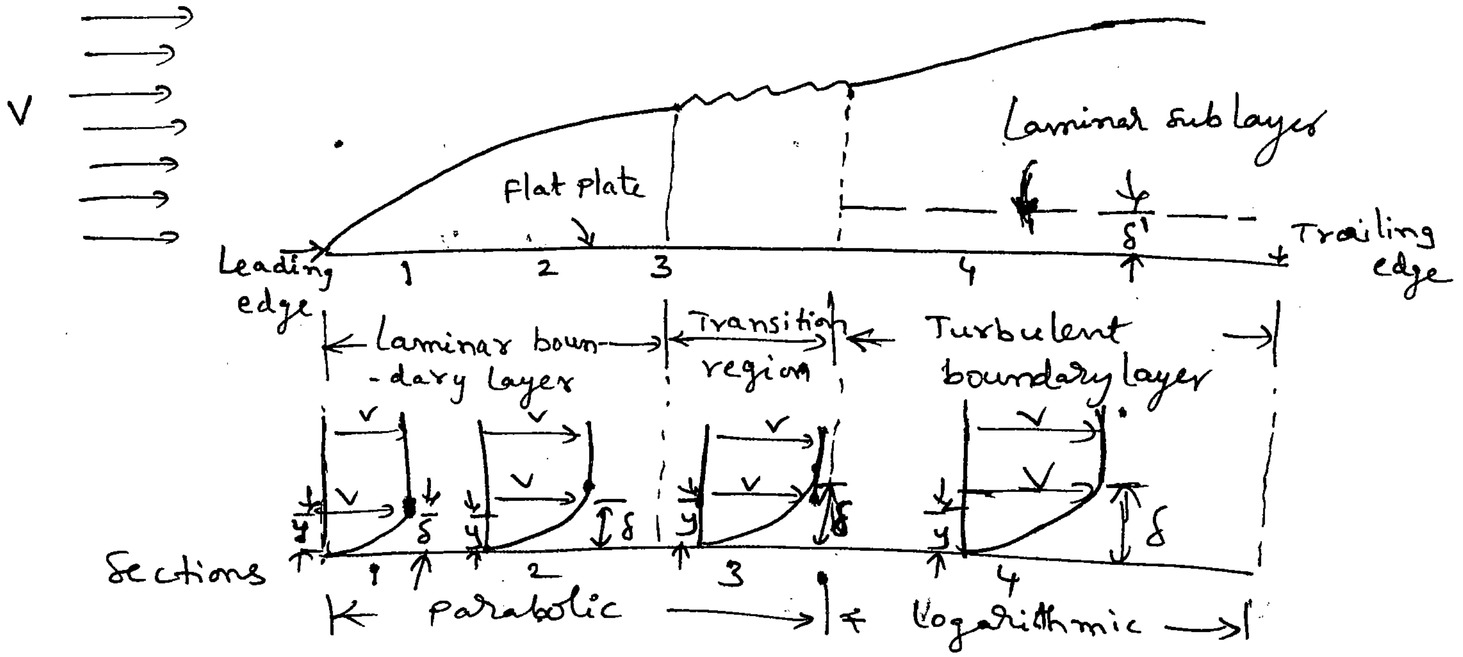
$$\frac{\partial V_z}{\partial z} = 0 \quad \text{which means } V_z = V_z\,(r, t).$$

⇒ Invoking, $\left[V_r = 0,\ V_\theta = 0,\ \frac{\partial V_z}{\partial z} = 0 \text{ and } \frac{\partial}{\partial \theta}(\text{any quantity}) = 0 \right]$ in the navier stokes equation, we finally obtain

$$\frac{\partial V_z}{\partial t} = -\frac{1}{\rho} + \nu \left[\frac{\partial^2 V_z}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial V_z}{\partial r} \right]$$

2) Differentiate between Laminar & Turbulent Boundary layer

S.No	Laminar Boundary layer	S.No	Turbulent Boundary Layer
1.	Velocity distribution follows parabolic law.	1.	Velocity distribution follows Logarithmic Law.
2.	Thickness of boundary layer is small and doesn't change rapidly.	2.	Thickness of boundary layer is large and changes rapidly.
3.	Velocity gradient is not steep.	3.	Velocity gradient is much steeper.
4.	Velocity distribution is not uniform	4.	Velocity distribution is more uniform.
5.	shear stress is comparatively small.	5.	shear stress is large



FM Mid - II Model Paper P. Siva Sankar M. Tech (SE)

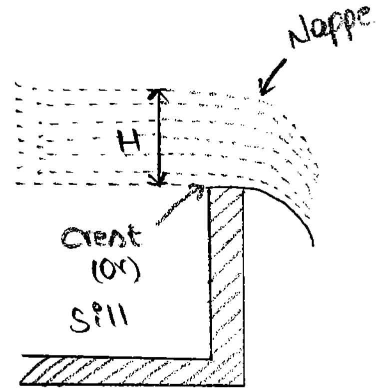
1. What are the different types of notches? Explain Rectangular and stepped notches?

Ans Types of notches:-

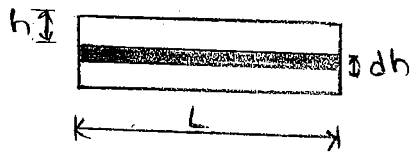
1. According to the shape of the opening:-
   - i) Rectangular notch
   - ii) Triangular notch
   - iii) Trapezoidal notch
   - iv) Stepped notch.

2. According to the effect of the sides on the nappe:
   - i) Notch with end contraction
   - ii) Notch without end contraction.

Rectangular notch:-



Rectangular notch



Section at crest.

Let us consider a rectangular notch provided in a channel carrying a water.

Let,
H = Head of water over the crest

h = Depth from the free surface

dh = thickness of elementary horizontal strip of water

Area of strip = $L \times dh$

theoretical velocity of water flowing through strip $= \sqrt{2gh}$.

Discharge through strip.

$$dQ = C_d \times \text{Area of strip} \times \text{theoretical velocity}$$

$$= C_d \times L \times dh \times \sqrt{2gh} \rightarrow (1).$$

Where, C_d = coefficient of discharge.

∴ The total discharge for the whole notch is determined by integrating equation - (1) with limits 0 to H.

$$Q = \int_0^H C_d \cdot L \cdot \sqrt{2gh} \cdot dh$$

$$= \cancel{\int} C_d \cdot L \cdot \sqrt{2g} \int_0^H h^{1/2} \cdot dh$$

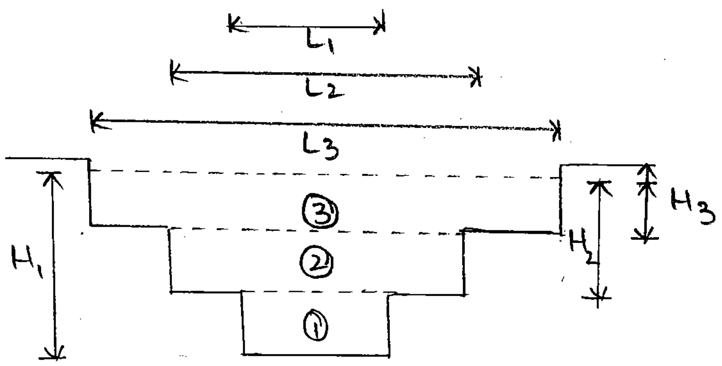
$$= C_d \cdot L \cdot \sqrt{2g} \left[\frac{h^{1/2+1}}{1/2+1} \right]_0^H$$

$$= C_d \cdot L \cdot \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_0^H$$

$$= \frac{2}{3} C_d \cdot L \cdot \sqrt{2g} \, [H]^{3/2}$$

Stepped notch:-

A stepped notch is a combination of rectangular notches. The discharge through stepped notch is equal to the sum of the discharges through the different rectangular notches.



consider a stepped notch as shown in the figure.

Let, H_1 = Height of water above the crest of notch - 1

L_1 = Length of notch-1.

H_2, L_2 & H_3, L_3 are corresponding values for notches 2 & 3 respectively.

C_d = coefficient of discharge for all notches.

Total discharge, $Q = Q_1 + Q_2 + Q_3$

$$Q = \frac{2}{3} \cdot C_d \cdot L_1 \cdot \sqrt{2g}\left[H_1^{3/2} - H_2^{3/2}\right] + \frac{2}{3} \cdot C_d \cdot L_2 \cdot \sqrt{2g}\left[H_2^{3/2} - H_3^{3/2}\right] + \frac{2}{3} \cdot C_d \cdot L_3 \cdot \sqrt{2g}\,\left[H_3\right]^{3/2}$$

2. Derive an expression for the loss of head due to friction in flow through circular pipes?

Sol Darcy's weisbach formula:-

The loss of head in pipes due to friction is calculated from Darcy's weisbach equation.

$$h_f = \frac{4fLV^2}{2gd}$$

Where, h_f = loss of head due to friction.

f = co-efficient of friction which is a function at Reynold's number.

L = Length of pipe

V = mean velocity of flow.

d = Diameter at pipe.

3. A venturimeter of throat diameter 5cm is fitted into a 12.5cm diameter water pipe line. The coefficient at discharge is 0.96. calculate the flow in the pipe line when the reading on a mercury water differential u-tube manometer connected to the upstream and sections shows a reading of 20cm.

Sol Given data,

Diameter of pipe, d_1 = 12.5 cm

Diameter of throat, d_2 = 5 cm

Area of pipe, $a_1 = \frac{\pi d^2}{4}$

$= \frac{\pi (12.5)^2}{4}$

$= 122.71\ cm^2$

Area of throat pipe, $a_2 = \frac{\pi d^2}{4}$

$= \frac{\pi (5)^2}{4}$

$= 19.63\ cm^2$.

Reading of differential u-tube manometer.

$x = 20\ cm$

venturi head, $h = x\left(\frac{S_m}{S_w} - 1\right)$

sp. gravity of mercury $= 13.6$

sp. gravity of water $= 1$

$h = 20\left(\frac{13.6}{1} - 1\right)$

$= 252\ cm$

coefficient of discharge, $C_d = 0.96$

Discharge through venturimeter is

$$Q = C_d \cdot \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$$Q = 0.96 \cdot \frac{122.71 \times 19.63 \sqrt{2 \times 9.81 \times 252}}{\sqrt{(122.71)^2 - (19.63)^2}}$$

$= 1342.36 \ cm^3/s$

$= 1.342$ lit/s.

4. Define velocity of approach. How can you account for it while computing the discharge over weirs?

Ans **Velocity of approach:-**

velocity of approach is defined as the velocity with which the water approach then additional head (h_a) equal to $\frac{V_a^2}{2g}$ due to velocity of approach. is acting on the water flowing over the notch. Then initial height of water over the notch become $(H+h_a)$ and final height become equal to h_a.

The velocity of approach, V_a is determined by finding the discharge over the notch neglecting of approach. Then dividing the discharge over the cross-sectional area of the channel on the upstream side of the notch. The velocity of approach is obtained, mathematically

$$V_a = \frac{Q}{\text{Area of channel}}$$

This velocity of approach is used to find an additional head $\left[h_a = \frac{V_a^2}{2g}\right]$. Again the discharge is calculated and above is used to process is repeated for more accurate discharge.

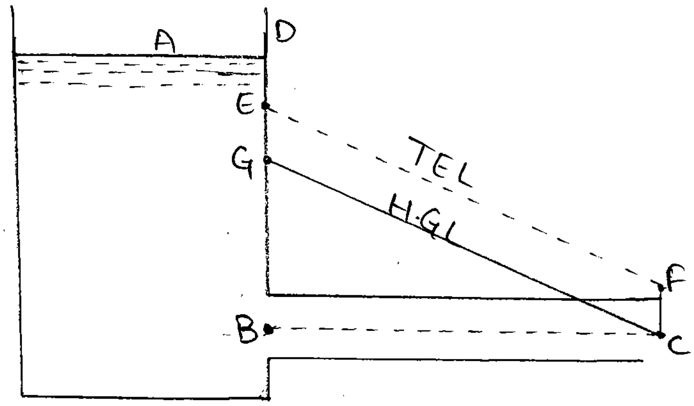
Discharge over a rectangular weir with velocity of approach.

$$= \frac{2}{3} \cdot C_d \cdot L \cdot \sqrt{2g} \left[(H_1 + h_a)^{3/2} - h_a^{3/2} \right]$$

5. what are hydraulic grade line and total energy line? How do you draw the same?

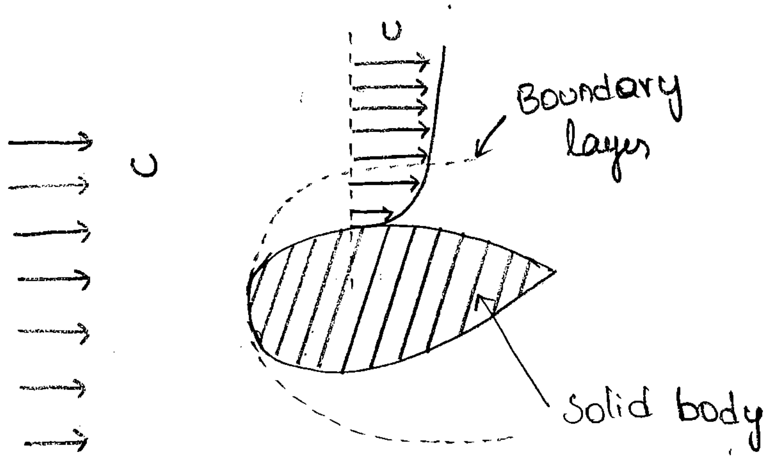
Ans Hydraulic Gradient line:- It is defined as the line which give the sum of pressure head $\left(\frac{P}{\omega}\right)$ and datum head (z) at a following fluid in a pipe with respect to some reference line or it is the line which is obtained by joining the top of all vertical ordinates, showing the pressure head $\left(\frac{P}{\omega}\right)$ at a following fluid in a pipe from the centre of the pipe. It is briefly written as H.G.L

Total energy line:- It is defined as the line which gives the sum of pressure head, datum head of a following fluid in a pipe with respect to some reference line. It is also defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of pressure head and kinetic head from the centre of the pipe. It is briefly written as T.E.L.



6. Define Boundary layer and explain the fundamental causes at its existence. Also discuss the various methods of controlling the boundary layer?

Ans Boundary layer:- consider the flow of a fluid having free stream velocity (U) over a smoothe. the plate which is a flat and placed parallel to the direction for free stream of fluid.



Flow over solid body.

Fundamental causes:-

→ A very thin layer of fluid, called the boundary layer in the immediate neighbourhood of the solid boundary.

→ The variation of velocity from zero at the solid boundary to free-stream velocity in the direction normal to the boundary takes place.

→ In this region, the velocity gradient $\frac{du}{dy}$ exists and hence the fluid exerts a shear stress on the wall in the direction of motion.

Methods

→ Suction of the slow moving fluid by a suction slot.

→ Supplying additional energy from a blower.

→ providing a bypass in the slotted wing.

→ Rotating boundary in the direction of flow.

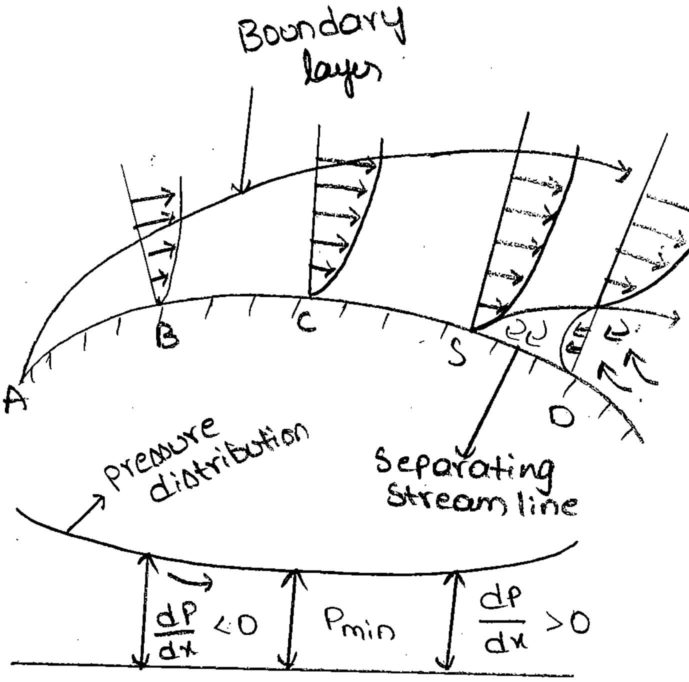
→ providing small divergence in a diffuser.

→ providing guide-blades in a bend.

→ providing a trip-wire ring in the laminar region for the flow over a sphere.

7. Explain boundary layer separation with a neat sketch. what are the conditions under which separation takes place?

Ans



when a solid body is immersed in a flowing fluid, a thin layer of fluid called the boundary layer is formed adjacent to the solid body. In this layer of fluid, the velocity varies from zero to free stream velocity in the direction normal to the solid body. Along the length of the solid body, the thickness of the boundary layer increases.

conditions under separation of Boundary layer:-

→ The effect of pressure gradient $(\frac{dp}{dx})$ on boundary layer separation can be explained by considering the flow.

→ The area of flow decreases and hence velocity increases.

→ Due to the increase of the velocity, the pressure decreases in the direction of flow.

→ pressure gradient $\frac{dp}{dx}$ is negative in this region.

→ As long as $\frac{dp}{dx} < 0$, the entire boundary layer moves forward.

8. A compound piping system consists of 1600m of 0.4m diameter, 1200m of 0.3m diameter and 800m pipe of 0.25m diameter cast iron pipes connecting in series. convert the system to i) an equivalent length of 0.4m pipe and ii) an equivalent size 3000m long.

Sol: Given data,

Length of pipe - 1 = 1600m

Length of pipe - 2 = 1200m

Length of pipe - 3 = 800m

Diameter of pipe - 1 = 0.4m

Diameter of pipe - 2 = 0.3m

Diameter of pipe - 3 = 0.25m.

Length of the single pipe, L = 3600 m.

Diameter of single pipe, d = ?

$$\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

$$\frac{3600}{d^5} = \frac{1600}{(0.4)^5} + \frac{1200}{(0.3)^5} + \frac{800}{(0.25)^5}$$

$$d^5 = \frac{3600}{1469277.16}$$

$$d = (0.00245)^{0.2}$$

$$= 0.3\,m$$

i) an equivalent length of 0.4m pipe

$$\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

$$\frac{L}{(0.4)^5} = \frac{1600}{(0.4)^5} + \frac{1200}{(0.3)^5} + \frac{800}{(0.25)^5}$$

$$L = 1469277.16 \times 0.01024$$

$$= 1504\,m.$$

ii) an equalent size 3000m long.

$$\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

$$\frac{3000}{d^5} = \frac{1600}{(0.4)^5} + \frac{1200}{(0.3)^5} + \frac{800}{(0.25)^5}$$

$$d^5 = \frac{3000}{1469277.16}$$

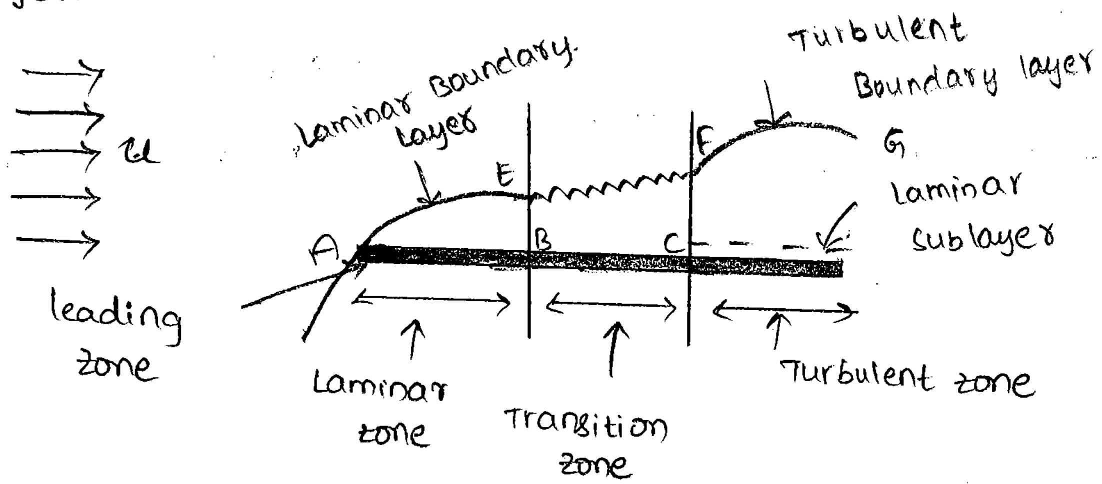
$$d = (0.002042)^{0.2}$$

$$d = 0.2\ m$$

9. what is a boundary layer? Differentiate between a laminar and turbulent boundary layer.

Boundary layer :-

When a fluid flows past a solid body or solid wall there is a variation of velocity from zero velocity at stationary Boundary to stream velocity in the direction normal to the Boundary taking in a narrow region in the vicinity of solid Boundary. The narrow region of Fluid is called "Boundary Layer."



FLOW OVER A PLATE

Laminar	Turbulent
* wear the leading edge where the thickness is small the flow is laminar. this layer is called Laminar Boundary layer	* It is downstream the Transition zone the Boundary layer is Turbulent where velocity gradient Increases.
* The length of the plate from leading Edge upto laminar Boundary layer is called laminar zone	* the length from Transitition boundary layer to Turbulent layer is called Turbulent zone.
* At laminar zone the shear stress is zero no back flow occurs	* At Turbulent zone the shear stress have some value which the back flow occurs.
* In laminar zone velocity is High, Area is low, Pressure is low	* In Turbulent zone Velocity is low, Area is High, pressure is High
* Reynolds number equal to 5×10^5 $(Re)_x = \frac{ux}{v}$ u = free stream velocity of fluid v = kinematic viscosity x = Distance from lead edge	* Reynolds number between 10^5 to 10^7 $\delta = \frac{0.37x}{(Re_x)^{1/5}}$ x = Distance from leading edge Re_x = Reynolds number for length 'x'