UNIT - III

FLUID DYNAMICS

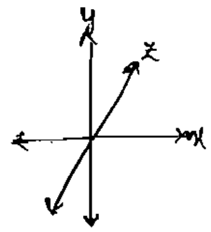
Fluid Dynamics:-

The branch of fluid mechanics which deals with the behaviour of the fluid in motion by considering the forces causing the motion fluid dynamics analyse the fluid motion by Newton's second law ($F = ma$)

$$F_x = ma_x$$

$$F_y = ma_y$$

$$F_z = ma_z$$



Types of forces acting on the fluid (body):-

- Gravity force
- pressure force
- Force due to viscosity
- Force due to Turbulence
- Force due to compressibility

If $\quad F = F_g + F_p + F_v + F_T + F_c$

$$\therefore \quad F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_T)_x + (F_c)_x$$

$$\therefore F_y = (F_g)_y + (F_p)_y + (F_v)_y + (F_T)_y + (F_c)_y$$

$$\therefore F_z = (F_g)_z + (F_p)_z + (F_v)_z + (F_T)_z + (F_c)_z$$

Condition-1 :-

If force due to compressibility is neglegible, the resultant equation is known as Reynold's equation.

$$(F_c)_x = 0$$

$$(F_c)_y = 0$$

$$(F_c)_z = 0$$

condition-2 :-

If force due to turbulance is neglegible the resultant equation is known as Navier stokes equation

$$(F_t)_x = 0$$

$$(F_t)_y = 0$$

$$(F_t)_z = 0$$

condition-3 :-

If the flow is assumed to be ideal. viscous forces are neglegible, the resulting equation is known as Euler's equation.

$$(F_v)_x = 0$$

$$(F_v)_y = 0$$

$$(F_v)_z = 0$$

Derivation of Euler's equation:

The equation of motion in which process due to gravity and force due to pressure are taken into consideration is known as Euler's equation.

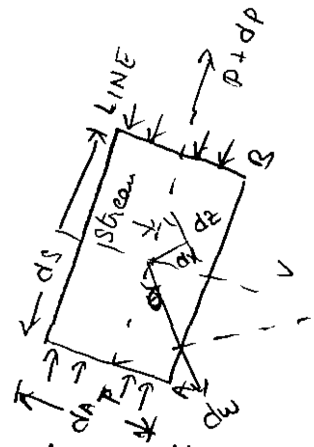
It is based on Newton's second law of motion ($F = ma$)

By integrating Euler's equation Bernoulli's equation is obtained.

Assumptions made in Euler's equation:

- Fluid is assumed as homogeneous (i.e., Density is constant) and incompressible.
- The fluid is ideal (i.e, non-viscous) force due to viscosity is neglegible.
- The velocity of flow is uniform over the section.
- No energy force ~~(except)~~ except gravity and pressure forces involved in the flow.

Derivation:



consider a steady flow of an ideal fluid along a stream line. Now, consider a small element AB of the flowing fluid as shown in figure.

Let, dA = cross-sectional area of the fluid element.

ds = Length of the fluid element.

dw = wt. of the fluid element.

p = pressure on the element ~~of A.~~ at A.

p + dp = pressure on the element at B.

Let v = velocity of the fluid element.

θ = Angle b/n the direction of flow and line of action of the wt. of the element.

we know the Euler's equation is based on Newton's ~~and~~ second law of motion

$$\therefore F = ma$$

$$F = ma$$

$$= \rho . dA . \cancel{ds} . v \frac{dv}{\cancel{ds}}$$

$$= \rho . v . dA . dv$$

$m = \rho \times v$
$= \rho \times dA \times ds$

$a = \frac{dv}{dt}$
$= \frac{dv}{ds} . \frac{ds}{dt}$
$= v . \frac{dv}{ds}$

∴ The net force acting on the fluid element.

$$= p . dA - (p + dp) dA - dw . \cos\theta$$

$$= p . dA - (p + dp) dA - \rho g . dA . \cancel{ds} . \frac{dz}{\cancel{ds}}$$

$$= - dp dA - \rho g dA . dz$$

$$F = \rho . v . dA . dv$$

$dw = w \times v$
$dw = w . dA . ds$
$dw = \rho . g . dA . ds$
$\cos\theta = \frac{dz}{ds}$

$$\therefore \quad - dp\, dA - \rho g . dA . dz = \rho . v . dA . dv$$

Dividing both sides by " $- \rho . dA$ "

$$+ \frac{dp}{\rho} + g . dz = - v . dv$$

$$\therefore \frac{dp}{\rho} + g . dz + v . dv = 0$$

The above equation is called Euler's equation.

Integration of Euler's equation is the Bernoulli's equation.

$$\frac{1}{\rho} \int dp + g . \int dz + \int v . dv = \int 0$$

$$\frac{p}{\rho} + g . z + \frac{v^2}{2} = \text{constant}$$

Multiplying with ρ on both sides, we get

$$p + \rho . g . z + \frac{\rho v^2}{2} = \text{constant}$$

$$p + w . z + \frac{w}{g} . \frac{v^2}{2} = \text{constant}$$

Dividing the entire equation by w.

$$\frac{p}{w} + z + \frac{v^2}{2g} = \text{constant}$$

$$\boxed{\frac{p}{w} + z + \frac{v^2}{2g} = \text{Constant}}$$

$z \rightarrow$ Datum head

The above equation is Bernoulli's equation for section 1 and 2, we get

$$\frac{p_1}{w} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{w} + z_2 + \frac{v_2^2}{2g} = \text{constant.}$$

Bernoulli's equation states that in a steady ideal and incompressible fluid the total energy remains constant at every point along the path of the flow.

Assumptions made in Bernoulli's equation:

- The fluid is ideal (viscous forces / Frictional forces are ~~negleg~~ neglegible).
- The liquid is incompressible and density is constant.
- The flow is steady (discharge constant).
- Flow is uniform across the cross section. (discharge constant)
- No other force ~~concep~~ except gravitational force should act on a liquid.

$= -1072 - 16633.836$

$= -17191.4 \ N$

$F = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{(18607.70)^2 + (17914)^2}$

$= 25.82 \ kN$

$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left[\frac{17914}{18607.7}\right]$

$= 43^\circ 54'$

Ideal - Non - Viscous

Real - Viscous

* Bernoulli's Equation to real fluids:-

Bernoulli's Equation derived based on the assumption that.

1. The fluid is ideal (Non - Viscous) shear forces and frictional forces are neglected but in practical all are real fluids. In the case of real real fluids when the flow takes place from one section to

another section loss of energy takes place. The loss of energy takes place. The loss of energy should be taken in to consideration in Bernoulli's Equation

The Bernoulli's equation for real fluids can be expressed as

1. $$\frac{P_1}{\omega} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\omega} + \frac{V_2^2}{2g} + z_2 + h_L$$

(Flow takes place from section ①-① to ②-②)

2. $$\frac{P_1}{\omega} + \frac{V_1^2}{2g} + z_1 + h_L = \frac{P_2}{\omega} + \frac{V_2^2}{2g} + z_2$$

(Flow takes place from section ②-② to ①-①)

* Problems:-

1. A pipe line carrying oil of specific gravity 0.87 changes in diameter from 200mm at position 'A' to 500mm at position B which is 4m at higher level. If the pressure at A and B are 9.81 N/cm^2 and 5.886 N/cm^2 respectively and the discharge is 200 lit/sec. Determine the loss of head and direction of flow.

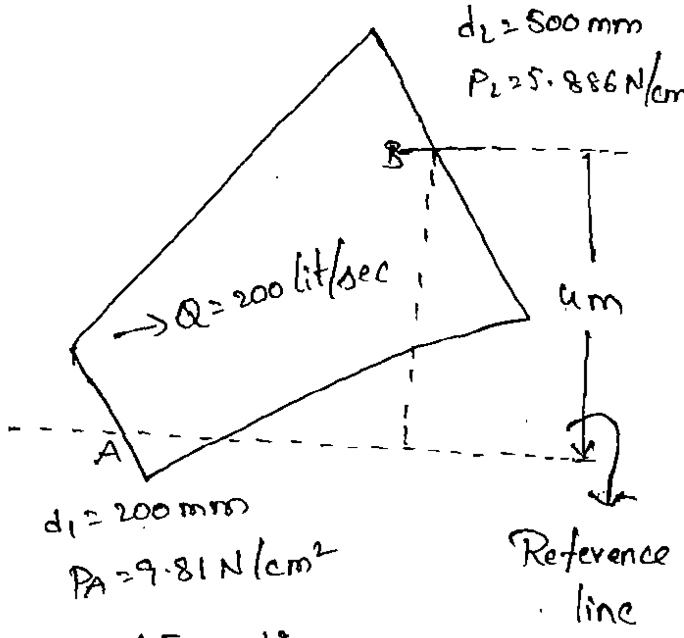
sol :-

$$G = \frac{P_{oil}}{P_w}$$

$$P_{oil} = 0.87 \times 100$$

$$w = \rho . g$$

$$\therefore w = 0.87 \times 1000 \times 9.81$$



$$Q = A_1V_1 = A_2V_2$$

$$\frac{200}{1000} = \frac{\pi}{4} \times \left(\frac{200}{1000}\right)^2 V_1 = \frac{\pi}{4}\left(\frac{500}{1000}\right)^2 V_2$$

$$\therefore V_2 = 1.018 \text{ m/sec}$$

$$\therefore V_1 = \frac{20}{3.14}$$

$$\therefore V_1 = 6.369 \text{ m/s}$$

$$E_A = \frac{P_A}{w} + \frac{v_A^2}{2g} + Z_A$$

$$= \frac{\cancel{9.81} \times 10^4}{0.87 \times \cancel{1000} \times \cancel{9.81}} + \frac{v_1^2}{2 \times 9.81} + 0$$

$$= \frac{10}{0.87} + \frac{(6.369)^2}{2 \times 9.81} = 13.56 m$$

$$E_B = \frac{P_B}{w} + \frac{v_B^2}{2g} + Z_B$$

$$= \frac{5.886 \times 10^4}{0.87 \times 1000 \times 9.81} + \frac{(1.018)^2}{2 \times 9.81} + 4$$

$$= 10.94 m$$

$\therefore E_A = 13.56\,m$ and $E_B = 10.94\,m$

Water always flows from higher energy to lower energy. Hence water flows from section A to section B.

$$E_A = E_B + h_2$$

$$\therefore h_2 = E_A - E_B = 13.56 - 10.94$$

$$= 2.62\,m$$

① To determine the resultant force acting on the boundary of a flow passage by the stream of fluid. As the stream changes its direction or magnitude of velocity or both.

The problems of these types are:

1. pipe bends
2. Stationary and moving vanes
3. Jet propulsion
4. Reducers etc...,

Limitations on Bernoulli's equation:

Bernoulli's theorem is derived based on some assumptions which are rarely possible. Hence it has the following limitations.

- Bernoulli's theorem is assumed that velocity is uniform across the section. But in real the velocity is maximum at centre and minimum at edges. The velocity is reduced to minimum at edges due to frictional force offered by the pipe material.
- Bernoulli's equation has been derived under the assumption that no external force ~~(accep)~~ except the gravitational force is acting on the liquid. But in practical some external forces such as pipe friction, centrifugal force when pipe takes curved path are acting on the fluid. Bernoulli's equation does not considered these external forces.
- Bernoulli's equation has been derived under the assumption that there is no loss of energy while fluid flowing. But in actual practise, some K.E ~~K.E~~ is converted into heat energy in turbulant flow. Hence there is loss of energy. In the case of viscous force there is some loss of energy.

due to the shear forces but the Bernoulli's equation neglects.

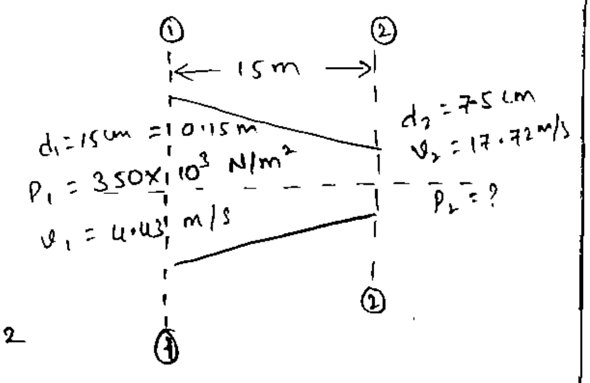
Use of Bernoulli's equation :-

Bernoulli's equation can be applied between the two points in order to determine the pressure & velocity at a section between two points. If pressure and velocity are known at the other section.

* Problems

1. Water flows in a pipe line at certain point where the diameter is 15 cm. Pressure and velocity 350 kN/m² and 4.43 m/s respectively. At a point 15 m away diameter of the pipe reduces to 7.5 cm If the flow occurs 15 cm to 7.5 cm. Find the pressure at 7.5 cm section. If the flow is a, Horizontal; b, Vertical.

Sol Horizontal.



$$A_1V_1 = A_2V_2$$

$$\frac{\cancel{\pi}(0.15)^2}{\cancel{4}}(4.43) = \frac{\cancel{\pi}(0.075)^2}{\cancel{4}} V_2$$

$$\therefore V_2 = 17.72\ m/s$$

$$\frac{p_1}{w} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{v_2^2}{2g} + z_2$$

Taking the centre of the pipe line as reference line $Z_1 = Z_2$

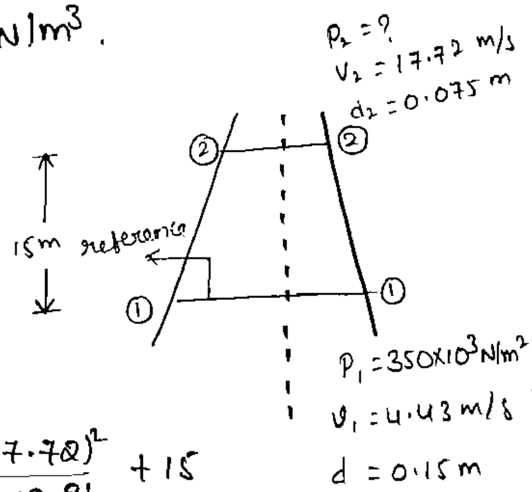
$$\therefore \quad \frac{P_1}{w} + \frac{v_1^2}{2g} = \frac{P_2}{w} + \frac{v_2^2}{2g}$$

$$\frac{350 \times 10^3}{9810} + \frac{(4.43)^2}{2 \times 9.81} = \frac{P_2}{9810} + \frac{(17.72)^2}{2 \times 9.81}$$

$$35.677 + 1.000 = \frac{P_2}{9810} + 16.003$$

$$P_2 = 202.77 \, kN/m^3.$$

<u>vertical</u>



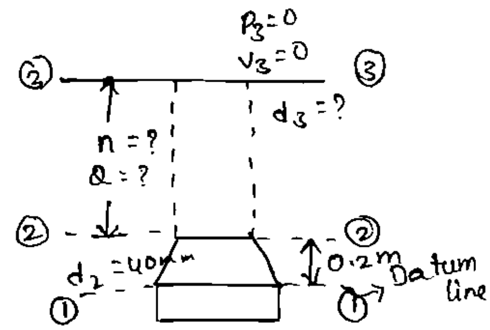
$$\frac{P_1}{w} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{w} + \frac{v_2^2}{2g} + Z_2$$

$$\frac{350 \times 10^3}{9810} + \frac{(4.43)^2}{2 \times 9.81} + 0 = \frac{P_2}{9810} + \frac{(17.72)^2}{2 \times 9.81} + 15$$

$$P_2 = 55.661 \, kN/m^3$$

3. A Jet of water issues vertically upwards from a 0.2 m high nozzle whose inlet and outlet diameters are 100mm and 40mm respectively. If the pressure at the inject is 20 kN/m² above the atmospheric pressure. Determine the discharge and the height to which the jet will rise.

Sol:- Applying Bernoullis Equation between section ① - ① and ② - ②



$d_1 = 100mm$

$P_1 = 20 kN/m^2$

$V_1 = ?$

$$\frac{P_1}{\omega} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\omega} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{20 \times 10^3}{9810} + \frac{V_1^2}{2g} + 0 = 0 + \frac{V_2^2}{2g} + 0.2$$

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = 1.838$$

Continuity Equation

$$A_1 V_1 = A_2 V_2$$

$$\frac{\pi}{4}\left(\frac{100}{1000}\right)^2 \times V_1 = \frac{\pi}{4}\left(\frac{40}{1000}\right)^2 \times V_2$$

$$\frac{100}{16} = \frac{V_2}{V_1}$$

$$\Rightarrow V_2 = 6.25\, V_1$$

$$\Rightarrow V_1 = 0.16\, V_2$$

$$\frac{1}{2g}\left[V_2^2 - (0.16)^2 V_2^2\right] = 1.838$$

$$V_2^2\,[0.9744] = 1.838 \times 2 \times 9.81$$

$$V_2 = 6.08 \text{ m/s}$$

$$V_1 = 0.9 \text{ m/s}$$

$$Q = A_1 V_1 = A_2 V_2$$

$$= \frac{\pi}{4} \times (100 \times 10^{-3})^2 \times 0.97$$

$$= 7.64 \times 10^{-3} \text{ m}^3/\text{sec}$$

Applying Bernoulli's Equation between section
2 - 2 and 3 - 3

$$\frac{P_2}{\omega} + \frac{v_2^2}{2g} + z_2 = \frac{P_3}{\omega} + \frac{V_3^2}{2g} + z_3$$

$$0 + \frac{(6.08)^2}{2 \times 9.81} + 0 = 0 + 0 + h$$

$$(\because z = \text{height})$$

$$\therefore \boxed{h = 1.88\,\text{m}}$$

→ Modification of Bernoulli's Equation to account for non-uniformity in velocity distribution & Kinetic Energy correction factor :-

Kinetic energy correction factor :-

Bernoull's Equation derived based on the assumption that velocity distribution is uniform across the section of pipe but in actual practice velocity distribution is maximum at centre and minimum at edges

The velocity head in Bernoulli's equation is effected by this assumption. Hence a correction factor is applied to the velocity head which is known as kinetic Energy correction factor.

It is defined as ratio of K.E based on Actual velocity to the K.E based on Average velocity.

$$K.E = \alpha = \frac{K.E/sec \text{ based on Actual velocity}}{K.E/sec \text{ based on Average velocity}}$$

$$\alpha = \frac{1}{A} \cdot \int \left(\frac{v_{act}}{\bar{v}_{avg}} \right)^3 dA.$$

After Applying the Bernoulli's Equation it can be expressed as

$$\frac{P}{w} + \alpha\left(\frac{v^2}{2g}\right) + z = k$$

In general $\alpha > 1$

$\alpha = 1$ Velocity distribution is uniform

$\alpha = 2$ laminar flow

$\alpha = 1.01$ to 1.2 for Turbulent flow

Momentum Equation :-

Momentum Equation is based on laws of the conservation of momentum it states that the net force acting on a fluid mass is equal to change in momentum of the flow per unit time in that direction

$$F = ma$$

$$a = \frac{dv}{dt}$$

$$F = m.\frac{dv}{dt}$$

$$F.dt = m.dv$$

$$F = \frac{d(mdv)}{dt}$$

(d)

Impluse momentum Equation states that the impulse of a force (F) acting on a fluid mass (m) in a short interval of time (dt) is equal to change in momentum of the fluid (d(mv))

$$\boxed{F \times dt = d(mv)}$$

→ Application of Impulse- momentum Equation Impulse-momentum Equation is used in the following two types of problem.

1) Pipe bends

2) Stationary & Moving Vanes

3) Jet propulsion

4) Reducers etc.

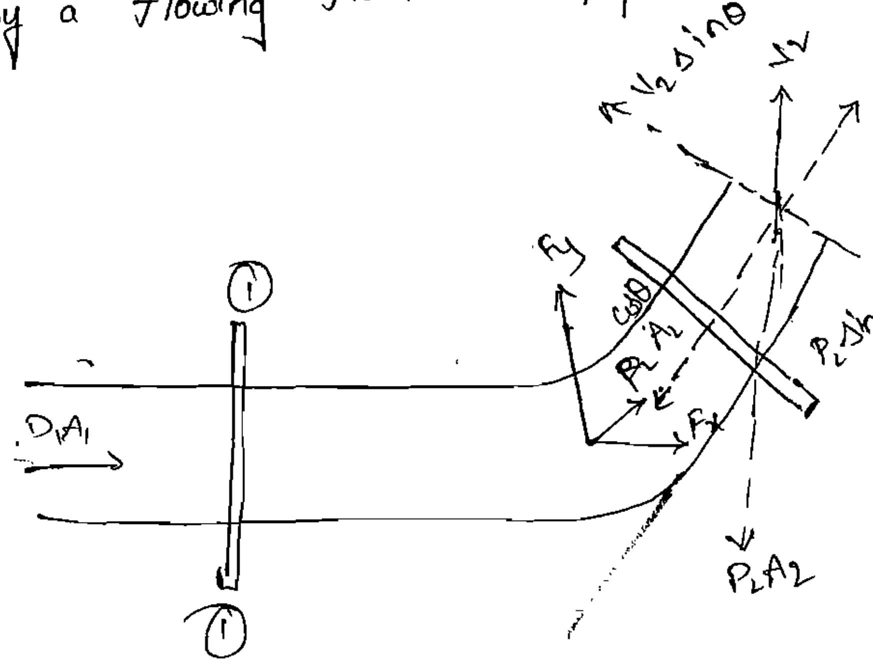
→ To determine the characteristics of flow when there is a abrupt change of flow section (non-uniform flow)

→ problems of this type includes

a) sudden enlargement in pipes

b) Hydraullic jump in the case of dam.

* → Force exerted by a flowing fluid on a pipe Bend



Consider 2 sections ① and ② as shown in figure

v_1 = Velocity of flow at section ①

P_1 = Pressure intensity at section ①

A_1 = Cross sectional area of pipe at section-①

$P_2 A_2 V_2$ = Corresponding value of Velocity, Pressure and area of section at ②.

→ Let F_x and F_y be the components of forces exerted by the flowing fluid on the bend in x and y directions respectively.

→ Then the force exerted by the bend on the fluid in the direction of x and y will be equal to F_x and F_y, but in opposite directions.

→ Hence the component of the force exerted by the pipe bend on fluid in x- direction $= -F_x$

In the direction of $y = -F_y$

→ the other external of y are acting on the fluid are P_1A_1 and P_2A_2 on the sections 1 and 2 respectively

→ The Applying Impulse momentum equation in x - direction

Net force = Mass per sec (change in Velocity)

$= \rho Q$

$$\boxed{F\,dt = M(V_2 - V_1)}$$

$$P_1 A_1 - P_2 A_2 \cos\theta - F_x = PQ(v_2 \cos\theta - v_1)$$

$$\boxed{F_x = P_1 A_1 - P_2 A_2 \cos\theta + PQ(v_1 - v_2 \cos\theta)} \longrightarrow (1)$$

Applying Impulse - Moment Equation in y-direction

$$0 - P_2 A_2 \sin\theta - F_y = PQ(v_2 \sin\theta - 0)$$

$$\boxed{F_y = PQ(-v_2 \sin\theta) - P_2 A_2 \sin\theta} \longrightarrow (2)$$

Resultant Force

$$F_R = \sqrt{F_x^2 + F_y^2}$$

$$\tan\theta = \frac{F_y}{F_x}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)}$$

In General :

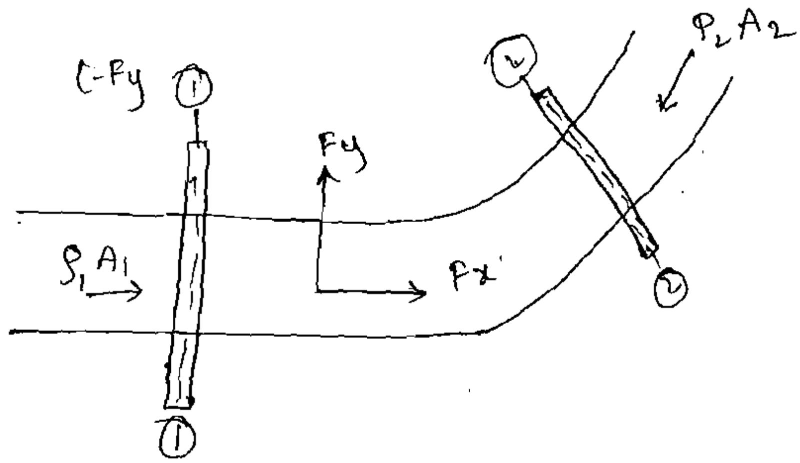
$$F_x = PQ(v_{1x} - v_{2x}) + P_1 A_1 - P_2 A_2 \cos\theta$$

$$F_y = PQ(v_{1y} - v_{2y}) - P_2 A_2 \sin\theta$$

* Problems On a Pipe Bend:-

$$F_x = PQ(v_{1x} - v_{2x}) + (P_1 A_1)_x - (P_2 A_2)_x$$

$$F_y = PQ(v_{1y} - v_{2y}) - (P_1 A_1)_y - (P_2 A_2)_y$$



* Problems:-

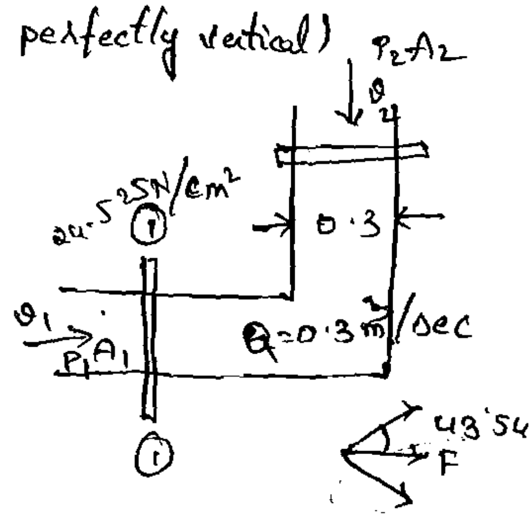
1. A pipe of 300mm diameter conleying 0.30 m^3/sec of water has a right angled bend in a horizontal plane. Find the resultant force entered by the bend if the pressure at inclined and out lined at the bend are 24.525 N/cm^2 and 23.544 N/cm^2

sol:-

(∵ it is perfectly vertical)

$$F_x = 1000 \times 0.30\,(4.24 - 0)$$

$$\frac{24.525 \times 10^4 \times \pi \times (0.3)^2}{4} - 0$$



$= 1272 + 17326.9125$

$= 18607.70N$

$Q = A_1 \times v_1$

$$v_1 = \frac{Q}{A_1} = \frac{0.3}{\frac{\pi(0.3)^2}{4}}$$

$$F_y = (1000 \times 0.3)(0 - 4.24) + 0 - \frac{23.544 \times 10^{-4} \times \pi(0.3)^2}{4}$$

$$= \frac{4.24}{v_2}\ m/sec$$

$= -1272 - 16633.836$

$= -171914\ N$

$F = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{(18607.70)^2 + (17914)^2}$

$= \sqrt{(18607.70)^2 + (17914)^2}$

$= 28.82\ kN$

$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left[\frac{17914}{18607.70}\right]$

$= 43°54'$

2. A 300m diameter pipe carries water under head of 20m with a velocity 3.5 m/s if the axis of the pipe turns through 45° magnitude and direction of resultant forces

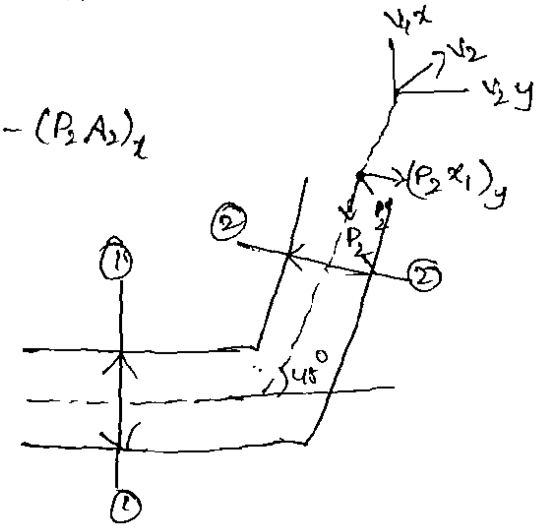
$F_x = \rho Q(V_{1x} - V_{2x}) + (P_1A_1)_x - (P_2A_2)_x$

$Q = A_1V_1 = A_2V_2$

$V_1 = V_2 = 3.5\ m/sec$

$Q = A\ \frac{\pi}{4}(0.3)^2 \times 3.5$

$= 0.247\ m^3/sec$



$P = wh$

$= 9810 \times 20$

$= 196200\ N/m^2$

$$F(x) = \rho Q\,(V_{1x} - V_{2x}) + (P_1A_1)_x - (P_2A_2)_x$$

$$= 1000 \times 0.247\,(3.5 - 3.5\cos 45^\circ) + \left(196200 \times \frac{\pi}{4}(0.3)^2\right) - \left(196200 \cos 45^\circ \cdot \frac{\pi}{4}(0.3)^2\right)$$

$$= 4310.69\,N$$

$$F(y) = \rho Q\,(\cancelto{0}{V_{1y}} - V_{2y}) - \cancelto{0}{(P_1A_1)_y} - (P_2A_2)_y$$

$$= 1000 \times 0.247\,(0 - 3.5\sin 45^\circ) - 0 - \left(196200 \sin 45^\circ \frac{\pi}{4}(0.3)^2\right)$$

$$= -10405.9\,N$$

$$F = \sqrt{(F_x)^2 + (F_y)^2}$$

$$= 11263.4\,N$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

$$= 67^\circ 29'$$