→ * Calculation of Deflection.

Problem: A rectangular cantilever beam of span 4M is 350 mm x 650 mm in cross section. Bending moment at the support due to UDL [Uniformly distributed loads] is 150 KN-m out of which 50% moment is due to permanent loads. Check the beam for deflection. It carries 3 25 mm dia bars in tension at an effective cover of 50 mm. Use M_{20} concrecte & HYSD steel.

Solution: Given data

* Span of the beam 'L' = 4M = 4000 mm.
* Width 'b' = 350 mm
* Overall depth 'D' = 650 mm.
* Effective cover 'd'' = 50 mm.
* Working Bending moment 'M' = 150 KN-m.
* Area of reinforcement 'A_{st}' = $3 \times \frac{\pi}{4} \times 25^2$
  = 3 x 491
  = 1473 mm^2.

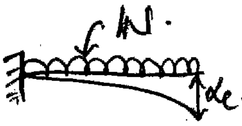
* percentage of moment due to permanent loads = 50%.

* f_{ck} = 20 mpa.
* f_y = 415 mpa.

→ Step: ① Calculation of Short term deflection: [clause - C-2]

$$\cancel{\alpha} = \cancel{\frac{Wl^4}{8EI}} = \cancel{\frac{W l^2}{2}\left[\frac{W l^2}{4EI}\right]}$$

$$\alpha_c = \frac{W \times L^4}{8EI} = \frac{W \times L^2}{2}\left[\frac{L^2}{4EI}\right] \Rightarrow \cancel{M \cdot \frac{L^2}{4EI}}$$



$$= M \cdot \frac{L^2}{4EI}.$$

Where $M = 150 \times 10^6$ N-mm.

$L = 4000$ mm.

$E_c = 5000\sqrt{f_{ck}}$. [clause: 6.2.3 in IS:456-2000]

$$I = I_{eff} = \frac{I_{cr}}{1.2 - \frac{M_{cr}}{M}\frac{z}{d}\left[1 - \frac{x}{d}\right]\frac{b_w}{b_f}}$$

$$I_{cr} = \cancel{bx}\ \frac{bx^3}{3} + m \cdot A_{st}\,[d-x]^2$$

$$m = \text{Modular ratio} = E_s / E_c = \frac{2 \times 10^5}{22360} = 8.94$$

To find neutral axis depth 'x'

$$\frac{b(x^2)}{2} = m \cdot A_{st}\,[d-x]$$

$$\Rightarrow \frac{350(x^2)}{2} = 8.94 \times 1473\,(600 - x]$$

$$\Rightarrow x = 178.16 \text{ mm}$$

Lever arm $z = d - \frac{x}{3} = 540.61$ mm.

$$\therefore I_{cr} = \frac{bx^3}{3} + m \cdot A_{st}\,[d-x]^2$$

$$= \frac{350 \times (178.16)^3}{3} + 8.94 \times 1473\,[600 - 178.16]^2$$

$$= 659.74 \times 10^6 + 2.34 \times 10^9$$

$$= 2.99 \times 10^9 \text{ mm}^4$$

$$I_{gr} = \frac{bD^3}{3} = \frac{350 \times 650^3}{12} = \cancel{3.2 \times 10^{10}} = 8 \times 10^9 \text{ mm}^4$$

$$f_{cr} = 0.7\sqrt{f_{ck}} = 3.13 \text{ mpa. } [\text{cl: } 6.2.2].$$

$$y_t = \frac{D}{2} = \frac{650}{2} = 325 \text{ mm.}$$

$$M_{cr} = \frac{f_{cr} \times I_{gr}}{y_t} = \frac{3.13 \times 8 \times 10^9}{325} = 77.04 \text{ KN-m.}$$

$$\frac{b_w}{b_f} = 1 \quad [\text{for rectangular section}]$$

$$\therefore I = I_{eff} = \frac{2.99 \times 10^9}{1.2 - \frac{77.04 \times 10^6}{150 \times 10^6} \times \frac{540.61}{600}\left[1 - \frac{178.16}{600}\right] \times 1}$$

$$= \frac{2.99 \times 10^9}{874.5 \times 10^{-3}}$$

$$I = I_{eff.} = 3.41 \times 10^9 \text{ mm}^4$$

$$\therefore \text{Deflection } \alpha_e = \frac{M \cdot L^2}{4 E_c I_{eff}}$$

$$= \frac{150 \times 10^6 \times 4000^2}{4 \times 22360 \times 3.41 \times 10^9}$$

$$\alpha_e = 7.86 \text{ mm.}$$

→ Step :② Calculation of shrinkage deflection: [cl: c-3]

$$\alpha_{cs} = k_3 \psi_{cs} L^2$$

Where $k_3 = 0.5$ for cantilever beam.

$L = 4000$ mm.

ψ_{cs} = Shrinkage curvature $= K_4 \frac{\varepsilon_{cs}}{D}$.

where $\varepsilon_{cs} = 0.0003$.

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$D = 650$ mm.

$K_4 = 0.72 \times \frac{P_t - P_c}{\sqrt{P_t}} \leq 1.0$

for $0.25 \leq P_t - P_c < 1.0$

$P_t = \frac{100 A_{st}}{bd} = \frac{100 \times 1473}{350 \times 600} = 0.7\,\%$

$P_c = 0$.

$\therefore \; K_4 = 0.72 \times \frac{0.7 - 0}{\sqrt{0.7}} = 0.60$

$\psi_{cs} = K_4 \frac{\varepsilon_{cs}}{D}$

$= 0.6 \times \frac{0.0003}{650} = 2.78 \times 10^{-7}$

$\therefore$ Shrinkage deflection

$\alpha_{cs} = K_3 \psi_{cs} L^2$

$= 0.5 \times 2.78 \times 10^{-7} \times 4000^2$

$= 2.22$ mm

→ Step : (3) Calculation of Creep Deflection [Cl : C-4]

$$\alpha_{cc\,(perm)} = \alpha_{i,\,cc\,(perm)} - \alpha_{i\,(perm)}$$

Where $\alpha_{i\,cc\,(perm)} = \dfrac{M.L^2}{4 E_{ce} I_{eff}}$

Where $E_{ce} = \dfrac{E_c}{1+\theta} = \dfrac{22360}{1+1.6}$ [Cl: 6.2.5.1] $\theta = 1.6$ for 28 days age of Concrete

$\Rightarrow E_{ce} = 8600$ mpa.

Then, Modular ratio $m = \dfrac{E_s}{E_{ce}} = \dfrac{2\times10^5}{8600}$

$\Rightarrow m = 23.25$

To find Neutral axis depth 'x'

$$\frac{bx^2}{2} = m.A_{st}\,[d-x]$$

$$\Rightarrow \frac{350\times x^2}{2} = 23.25\times1473\,[600-x]$$

$\Rightarrow x = 258.51$ mm

Lever arm $Z = d - \dfrac{x}{3} = 600 - \dfrac{258.51}{3}$

$\Rightarrow Z = 513.83$ mm

$$I_{cr} = \frac{bx^3}{3} + m.A_{st}\,[d-x]^2$$

$$= \frac{350\times258.51^3}{3} + 23.25\times1473\,[600-258.51]^2$$

$$= 2.01\times10^9 + \cancel{6.2.26}\ 3.99\times10^9$$

$I_{cr} = 6\times10^9\ mm^4$

$$I = I_{eff} = \frac{I_{cr}}{1.2 - \frac{M_{cr}}{M} \frac{Z}{d}\left[1 - \frac{x}{d}\right]\frac{b_W}{b_f}}$$

$$= \frac{6 \times 10^9}{1.2 - \frac{77.04}{150} \times \frac{513.83}{600}\left[1 - \frac{258.51}{600}\right] 1}$$

$$I = I_{eff} = 6.3 \times 10^9 \ mm^4$$

$$\therefore \alpha_{i(cc)(perm)} = \frac{M_{(perm)} \cdot L^2}{4 E_{ce} \times I_{eff}}$$

$$= \frac{0.5 \times 150 \times 10^6 \times 4000^2}{4 \times 8600 \times 6.3 \times 10^9}$$

$$\Rightarrow \alpha_{i(cc)(perm)} = 5.5 \ mm$$

$$\alpha_{i(perm)} = \frac{M_{(perm)} \cdot L^2}{4 E_{ce} \times I_{eff}}$$

$$= \frac{0.5 \times 150 \times 10^6 \times 4000^2}{4 \times 22360 \times 3.41 \times 10^9}$$

$$\Rightarrow \alpha_{i(perm)} = 3.93 \ mm$$

$$\therefore \alpha_{(cc)(perm)} = 5.5 - 3.93 = 1.57 \ mm$$

→ Step : (4) Calculation of Total Deflection [Cl: C-1]

Total Deflection 'α' = Shortterm deflection + long term deflection

$$= \alpha_{ie} + [\alpha_{cs} + \alpha_{cc}]$$

$$= 7.86 + [2.22 + 1.57]$$

$$\alpha = 11.65 \text{ mm}$$

—* Check for total deflection [Cl: 23.2.1]

* permissiable vertical deflection of a structural member of the age of 28 days = span/250

$$= 4000/250 = 16 \text{ mm}$$

$$> \alpha = 11.65 \text{ mm}$$

Hence beam is safe in deflection