UNIT-2

13.3 METHODS OF DEVELOPMENT

Solids bounded by plane surfaces and single curved surfaces can be developed by: (i) Parallel line development method, based on stretch-out line principle; used for prisms and cylinders, (ii) radial line development method, making use of true length of slant edge or generator; used for pyramids and cones, (iii) triangulation method normally used for developing the transition pieces - connecting ducts, pipes, openings and similar objects with various sizes and shapes, (iv) approximate method, used to develop the objects with double curved or warped surfaces such as sphere, paraboloid, ellipsoid, hyperboloid, etc.

Only the lateral surfaces are generally developed and shown as presented here, omitting the bases or ends of solids.

13.3.1 Parallel line development

The surfaces of right prisms, cylinders and also oblique prisms and cylinders may be developed by this method.

Problem 1 *A square prism of side of base 40 and axis 80 long, is resting on its base on H.P such that, a rectangular face of it is parallel to V.P. Draw the development of the prism.*

Construction (Fig. 13.1)

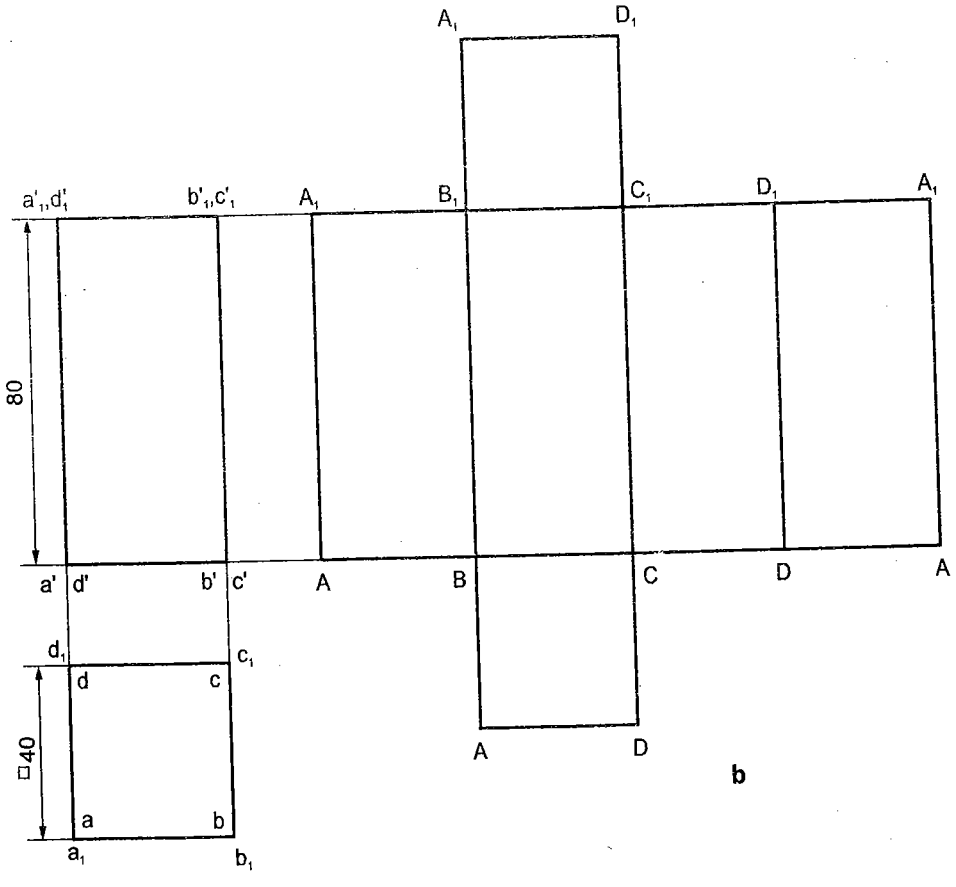


Fig.13.1

1. Draw the projections of the prism.
2. Draw the stretch-out line AA and mark - off the sides of the base along this line in succession, i.e., AB, BC, CD and DA.
3. Erect perpendiculars through these points and mark the edges AA_1, BB_1, etc.
4. Add the bases ABCD and $A_1B_1C_1D_1$ suitably.

NOTE (i) Stretch-out line is drawn in-line with the base in the front view, to complete the development quickly.

(ii) Generally, the lateral surfaces of the solids are developed and the bases are omitted.

(iii) All the lines on the development should represent the true lengths.

Problem 2 *A hexagonal prism of side of base 30 and axis 75 long, is resting on its base on H.P such that, a rectangular face is parallel to V.P. It is cut by a section plane, perpendicular to V.P and inclined at* 30° *to H.P. The section plane is passing through the top end of an extreme lateral edge of the prism. Draw the development of the lateral surface of the cut prism.*

Construction (Fig. 13.2)

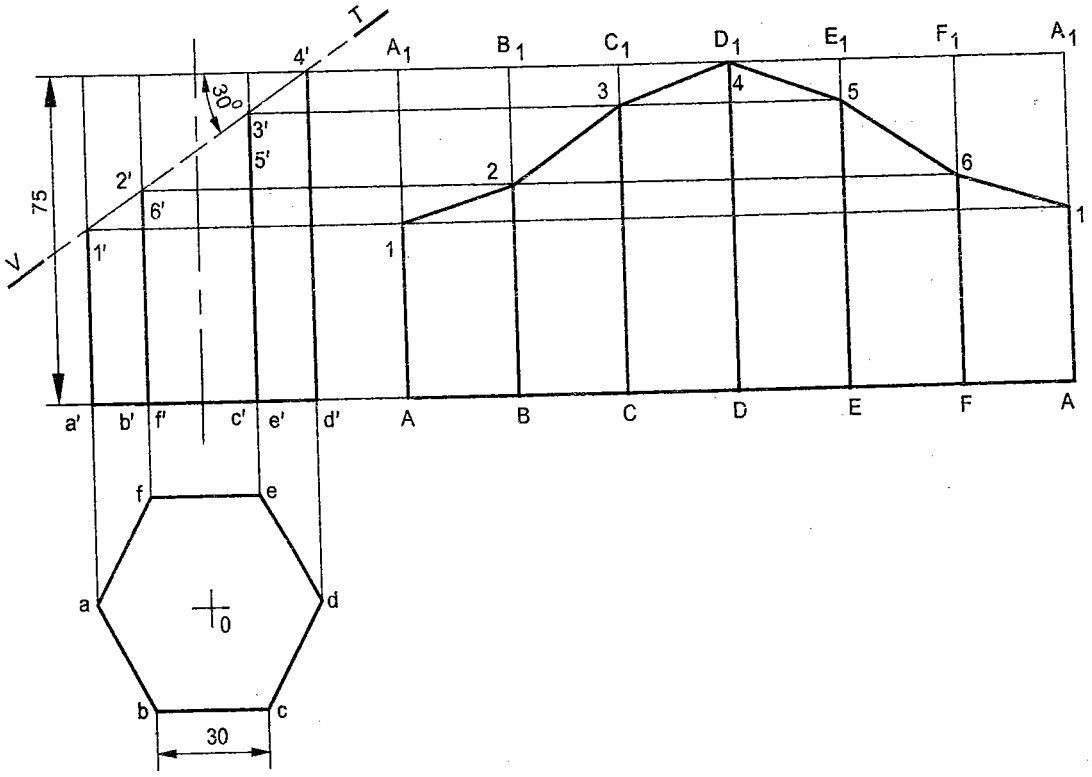


Fig.13.2

1. Draw the projections of the prism.
2. Draw the V.T of section plane, satisfying the given conditions.
3. Draw the development AA_1 - A_1A of the complete prism, following the stretch-out line principle.
4. Locate the points of intersection 1′, 2′, etc., between the V.T and edges of the prism.

5. Draw horizontal lines through 1′, 2′, etc., and obtain 1, 2, etc., on the corresponding edges in the development.
6. Join the points 1, 2, etc., by straight lines and darken the sides, corresponding to the retained portion of the solid.

NOTE It is the usual practice to cut open (for the development) the surface of the solid at the shortest edge/length.

Problem 3 *A cube of 50 edge, is resting on a face on H.P such that, a vertical face is inclined at 30° to V.P. It is cut by a section plane perpendicular to V.P and inclined to H.P at 30° and passing through a point at 12 from the top end of the axis. Develop the lateral surface of the lower portion of the cube.*

Construction (Fig. 13.3)

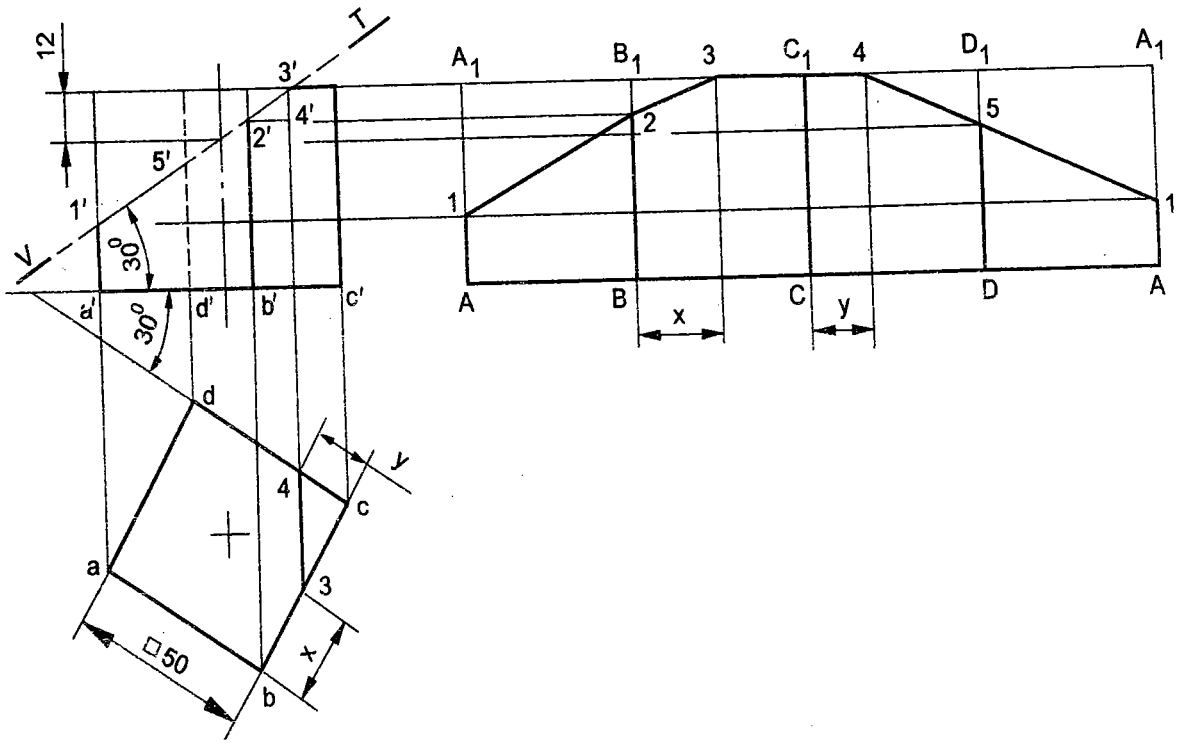


Fig.13.3

1. Draw the projections of the cube.
2. Draw the V.T of section plane, satisfying the given conditions.
3. Draw the development AA_1 - A_1A of the complete cube, following the stretch-out line principle.
4. Repeat steps 4 to 6 of Construction: Fig. 13.2 and obtain the development of the cut solid.

NOTE 1. The points 3′ and 4′ are on the top surface of the cube. To locate the corresponding points in the development:

(i) Draw a projector through 3′ (4′), meeting bc at 3 and cd at 4 in the top view.

(ii) Mark the points 3 and 4 in the development such that, $B_1 3 = b3 = x$ and $C_1 4 = c4 = y$.

2. The sectioned portion in the top view is not cross-hatched as it is made use of only for locating certain points in the development.

Problem 4 *A cylinder of diameter of base 40 and axis 55 long, is resting on its base on H.P. It is cut by a section plane, perpendicular to V.P and inclined at 45° to H.P. The section plane is passing through the top end of an extreme generator of the cylinder. Draw the development of the lateral surface of the cut cylinder.*

Construction (Fig. 13.4)

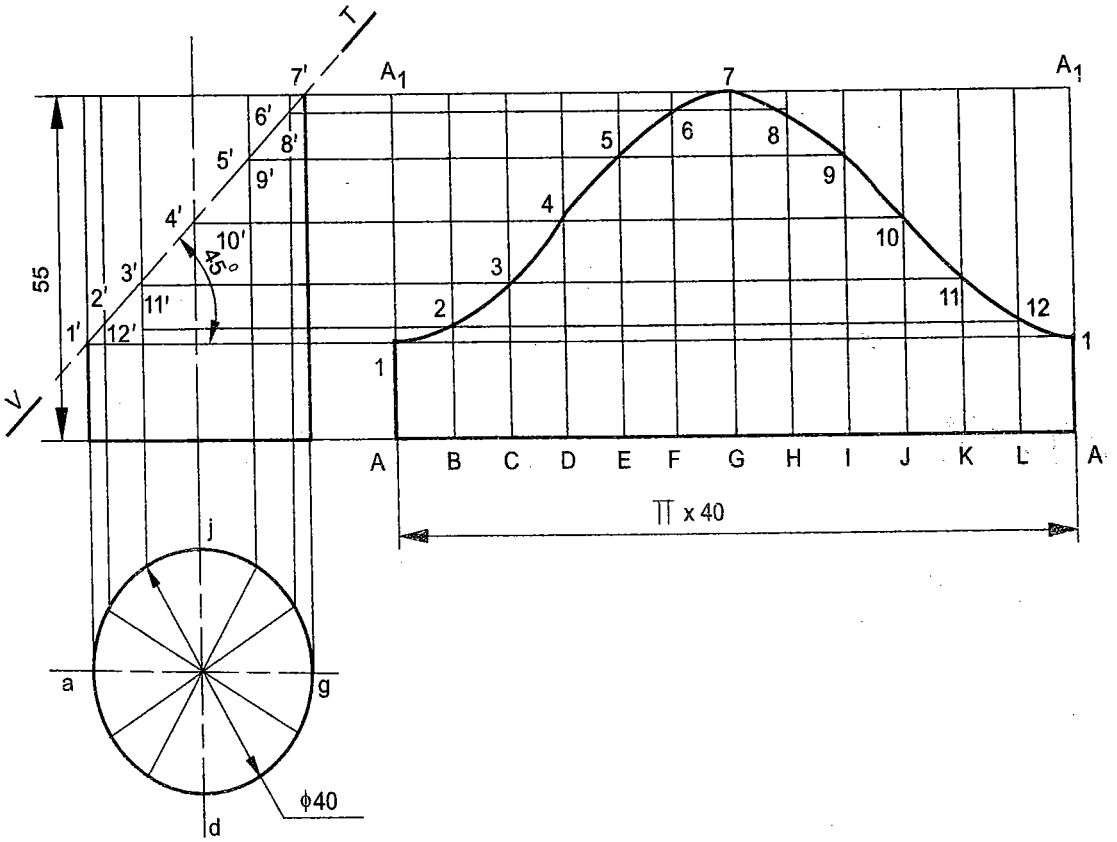


Fig.13.4

1. Draw the projections of the cylinder.
2. Divide the circle (top view) into a number of equal parts and locate the corresponding generators in the front view.
3. Draw the V.T of section plane, satisfying the given conditions.
4. Draw the stretch-out line AA, equal to the circumference of the base of the cylinder.
5. Divide the stretch-out line AA, into the same number of equal parts as that of the base circle/ set-off chord lengths by a divider and locate the generators through the division points B, C, D, etc.
6. Locate the points of intersection 1′, 2′, etc., between the V.T and generators.

7. Transfer these intersection points to the corresponding generators in the development, by projection.
8. Join the points 1, 2, etc., by a smooth curve and obtain the development.

NOTE (i) AA_1 - A_1A represents the development of the complete cylinder.

(ii) Only one half of the development may be shown when a solid is symmetric about an axis.

(iii) The generators should not be drawn thick, since they do not represent the folding edges.

Problem 5 *Figure 13.5a shows the projections of a cut cylinder. Draw the development of the lateral surface of the cut cylinder.*

Construction (Fig. 13.5)

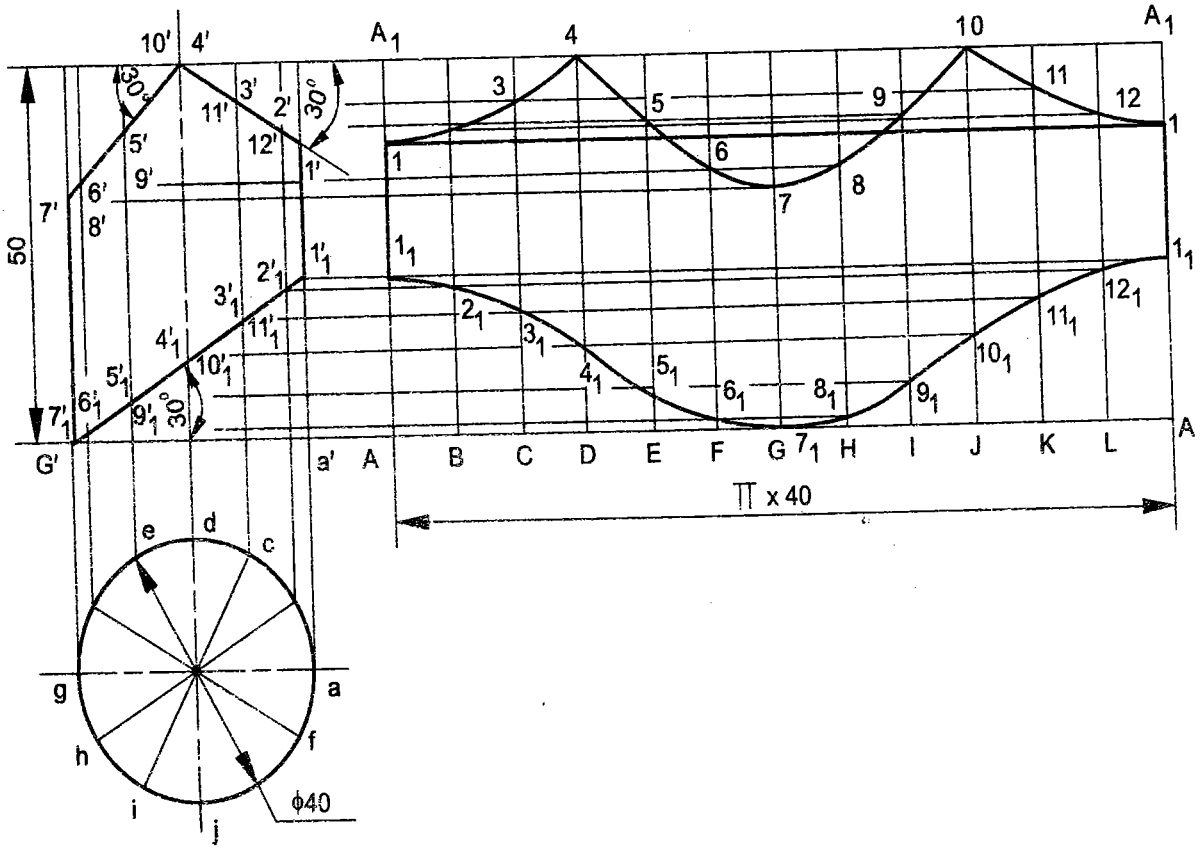


Fig.13.5

1. Draw the given projections of the cut cylinder.
2. Divide the base circle (top view) into a number of equal parts and locate the corresponding generators in the front view.
3. Draw the stretch-out line AA, equal to the circumference of the base and complete the development of the complete cylinder.
4. Locate the generators in the development.
5. Locate the points of intersection between the cut edges and generators.
6. Repeat steps 7 and 8 of Construction: Fig. 13.4 suitably and obtain the development of the cut cylinder.

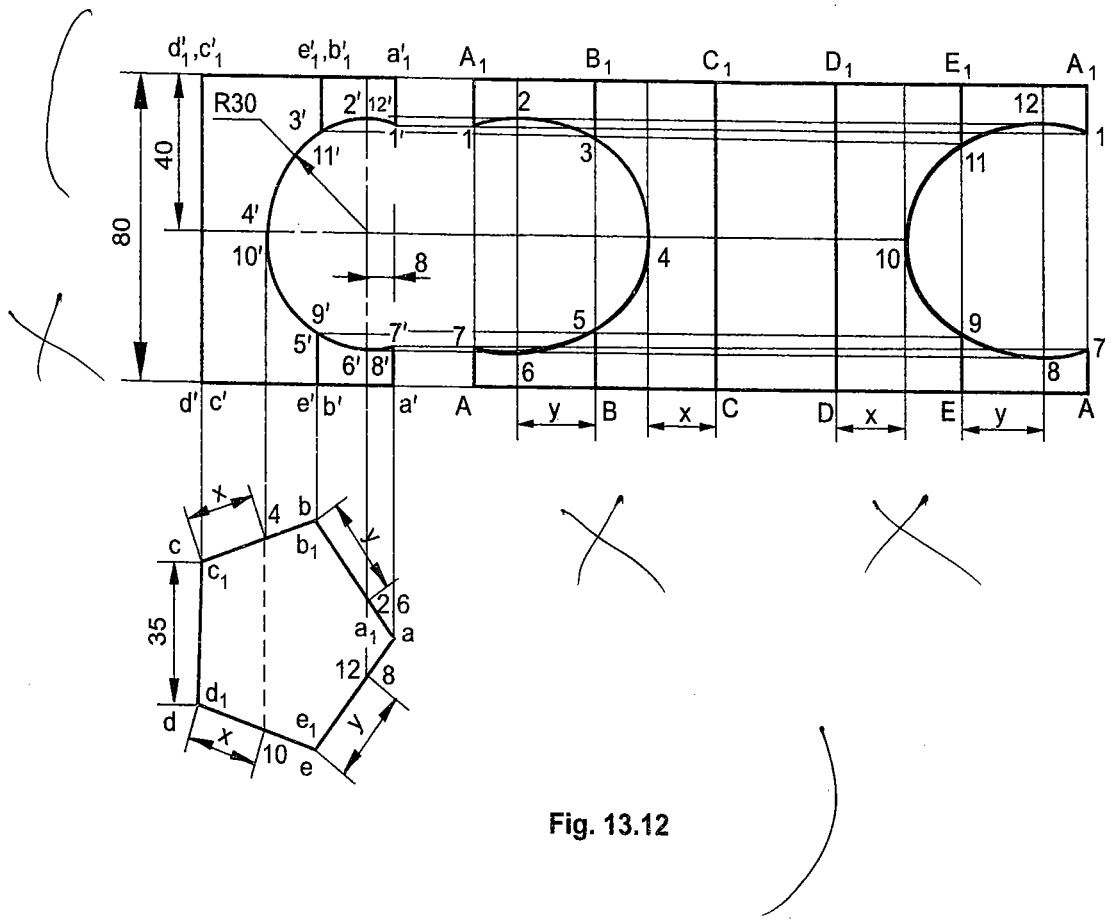


Fig. 13.12

Problem 13 *A hexagonal prism of 20 side of base and 50 height, rests on a base on H.P, with a vertical face parallel to V.P. A circular hole of 35 diameter, is drilled through the prism such that, the axis of the hole bisects the axis of the prism and is perpendicular to V.P. Draw the development of the prism.*

Construction (Fig.13.13)

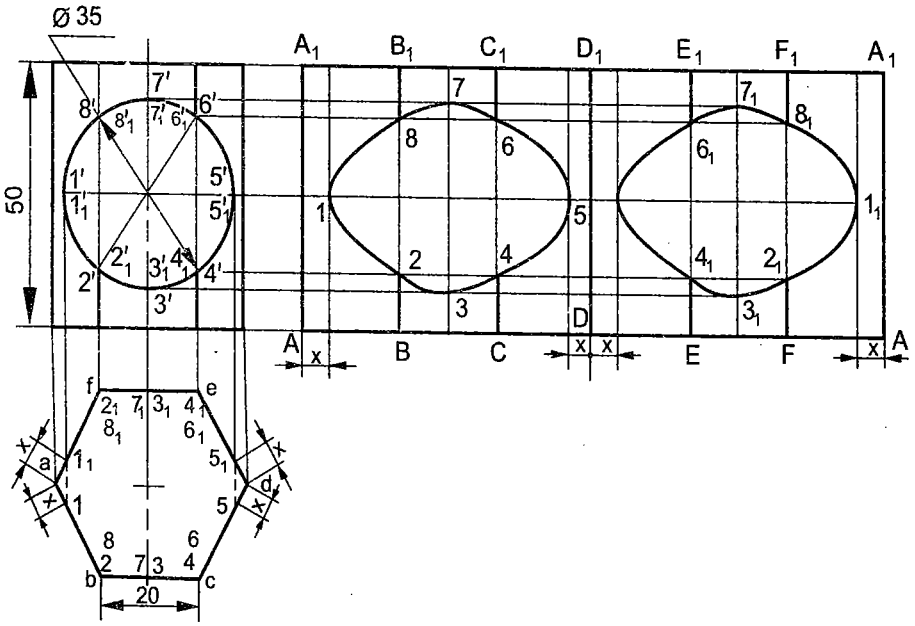
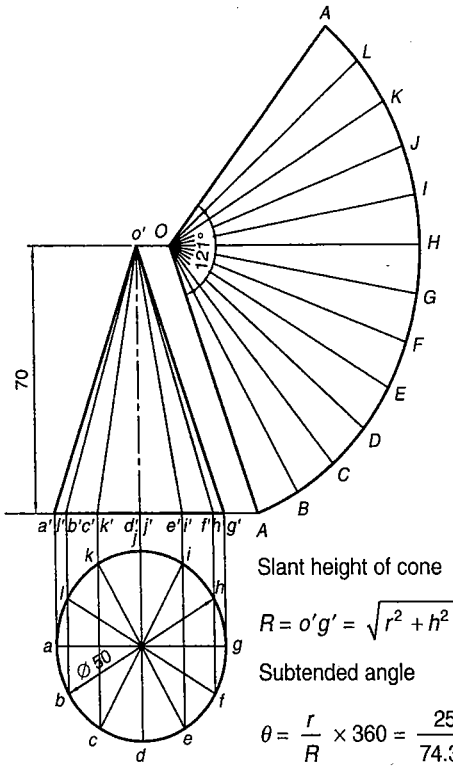


Fig. 13.13

PROBLEM 13.6 (FIG. 13.6) *Draw the development of lateral surface of the cone with a 50 mm base diameter and a 70 mm long axis. The cone is resting on H.P. on its base.*

CONSTRUCTION Figure 13.6

1. Draw the projections of the cone (a circle in the top view and a triangle $1'o'7'$ in front view). Draw the generators of the cone in its projections and name them as shown.
2. The end generators $o'1'$ and $o'7'$ give the true length of the generators because their top views are parallel to xy. Therefore, mark $O1$ parallel to $o'7'$.
3. Determine the subtended angle θ of the development.
4. Draw the sector 1-O-1 of radius $O1 = o'7'$ subtending included angle θ.
5. Divide sector into 12 equal parts and mark the generators as $O1$, $O2$, $O3$, etc. This is the required development of the cone.



Slant height of cone

$$R = o'g' = \sqrt{r^2 + h^2} = \sqrt{25^2 + 70^2} = 74.33 \text{ mm}$$

Subtended angle

$$\theta = \frac{r}{R} \times 360 = \frac{25}{74.33} \times 360° = 121° \text{ (approx.)}$$

Fig. 13.6 *Calculation of* θ

Note:

1. Before drawing the development of the truncated cone or frustum of the cone, the development of the complete cone has to be drawn using the above method.
2. Usually, 12 generators are drawn for the cone and cylinders. Generators are drawn light as they are the construction lines.

PROBLEM 13.7 (FIG. 13.7) *A cone with a 50 mm base diameter and a 70 mm long axis, rests on its base on the H.P. Draw the development of its lateral surface when it is cut by an auxiliary inclined plane bisecting the axis and inclined at 45° to the H.P.*

CONSTRUCTION Figure 13.7

1. Draw the projections of the complete cone and its development along with generators, as discussed in Problem 13.6.
2. Draw the cutting plane V.T. in the front view passing through mid-point of the axis and inclined at an angle of 45° to the xy line.
3. Locate the point of intersection of the cutting plane with the generators and name them as p', q', r', s', ..., etc.
4. Draw the horizontal lines from points p', q', r', s', ..., etc., to meet OA in the development at points p'', q'', r'', s'', ..., etc.
5. Draw arcs with O as the centre and radius Op'', Oq'', Or'', Os'', ..., etc., to meet the corresponding generators at points P, Q, R, S, ..., etc.
6. Join the points obtained with a smooth curve as shown. This is the required development.

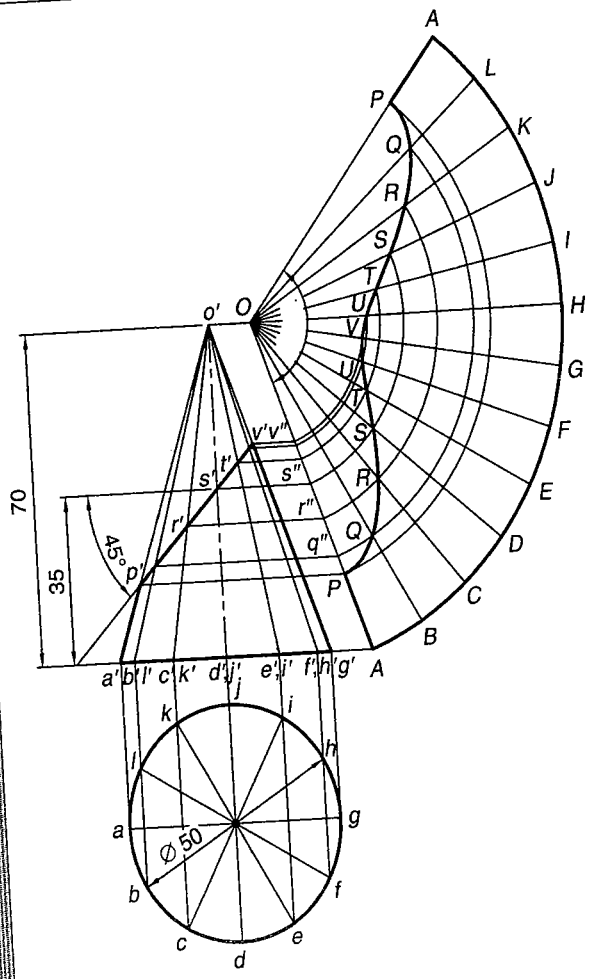


Fig. 13.7

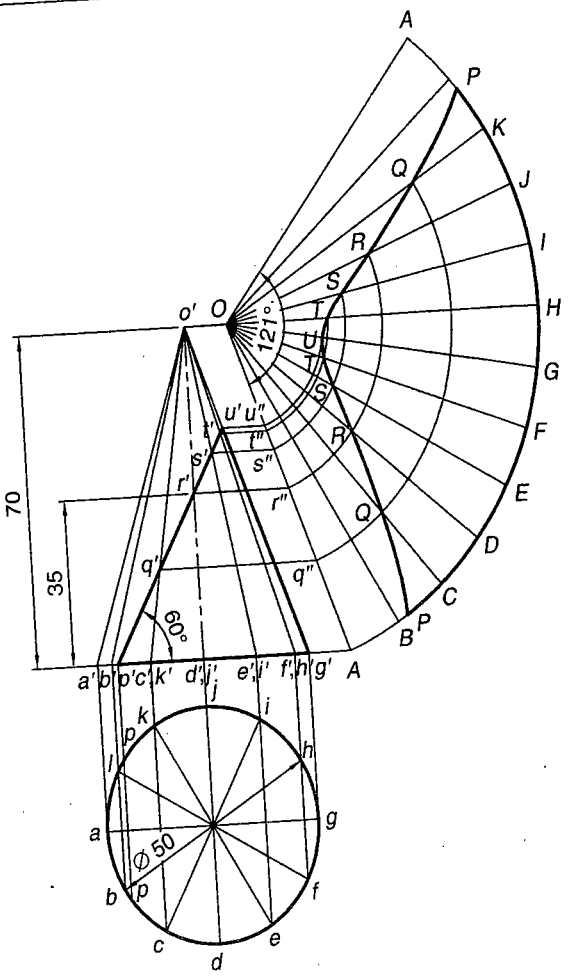


Fig. 13.8

PROBLEM 13.8 (FIG. 13.8) *A cone with a 50 mm base diameter and a 70 mm long axis, rests on its base on the H.P. Draw the development of its lateral surface when it is cut by an auxiliary inclined plane bisecting the axis and inclined at 60° to the H.P.*

CONSTRUCTION Figure 13.8

1. Draw the projections of the complete cone and its development as discussed in solved Problem 13.6.
2. Draw the cutting plane V.T. in the front view passing through mid-point of the axis and inclined at 60° with *xy*.
3. Locate the point of intersection of the cutting plane with the generators and name them as a', b', c', d', etc.
4. Draw the horizontal lines from points of intersection, i.e., b', c', d', e', f' which meets $O1$ at points a'', b'', c'', d'', e'', f''. With centre O and radius Ob'', Oc'', Od'', Oe'', Of'' draw arcs to meet the corresponding generators at points B, C, D, E, F.
5. Draw a vertical line from point a' meeting top view at points a. Transfer the distance 2-a and 12-a in the development to obtain point A so that 2-A = 2-a and 12-A = 12-a.
6. Join the points with a smooth curve as shown. This is the required development.

13.7 DEVELOPMENT OF PYRAMIDS

Development of lateral surface of pyramids is done by radial line method in a similar way as the cone. It may be remembered that the lateral surface of the pyramids is always an isosceles triangle which can be constructed if the true length of base and the slant edge is known. The true length of base side is directly available in the top view, but the true length of the slant edge may or may not be available in the front view. In case it is not available, we have to obtain true length of the slant edge first followed by usual procedure to obtain the development.

The following examples illustrate the development of lateral surface of pyramids.

PROBLEM 13.9 (FIG. 13.9) *Draw the development of lateral surface of a square pyramid with a 40 mm base side and a 60 mm long axis which is resting on its base in the H.P., when (a) all the sides of the base are equally inclined to the V.P., and (b) a side of the base is parallel to the V.P.*

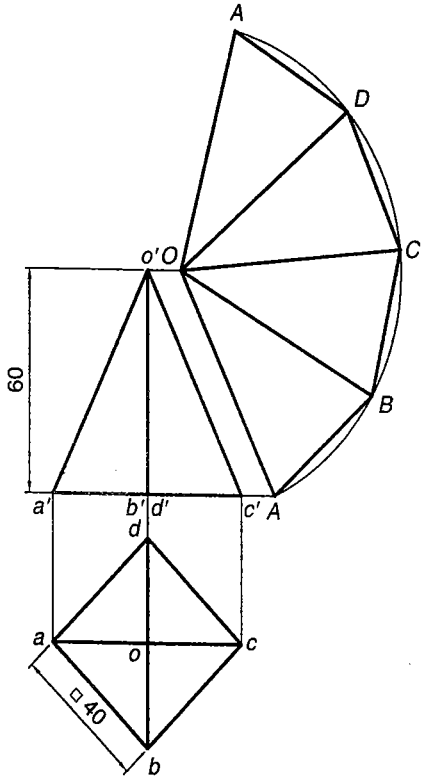


Fig. 13.9(a)

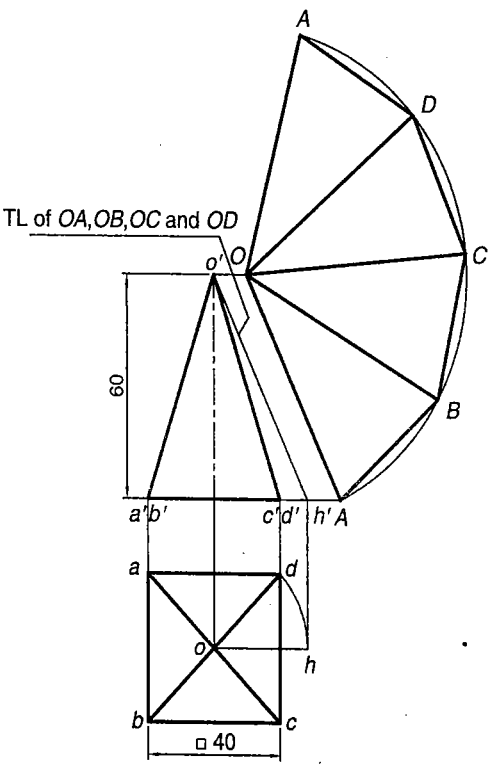


Fig. 13.9(a)

CONSTRUCTION When all the sides of the base are equally inclined to the V.P. Figure 13.9(a)

1. Draw a square *abcd* as the top view and project the corners to obtain triangle *a'o'c'* as the front view. Consider seam at *o'a'*.
2. The slant edges *o'c'* gives the true length because its top views *oc* is parallel to *xy*. Therefore, mark *OA* parallel to *o'c'*.
3. Draw an arc *A-A* with *O* as the centre and radius *OA*.
4. Step off the arc *A-A* into 4 divisions such that *AB* = *BC* = *CD* = *DA* = 40 mm and obtain points *A, B, C, D, A*.
5. Join base edges *AB, BC, CD, DA*. Also, join slant edges *OA, OB, OC, OD, OA*. This is the required development of the pyramid.

When a side of the base is parallel to the V.P. Figure 13.9(b)

1. Draw a square *abcd* as the top view and project the corners to obtain triangle $a'o'c'$ as the front view. Consider seam at $o'a'$.
2. Draw an arc *dh* with *o* as the centre and radius *od* to meet the horizontal line through centre *o* at point *h*.
3. Project point *h* to meet the *xy* line at point h'. Join $o'h'$. This $o'h'$ is the true length of slant edges because in the top views *oh* is parallel to *xy*.
4. Draw a line *OA* parallel and equal to $o'h'$, at an arbitary distance from $o'h'$.
5. Draw an arc *A-A* with *O* as the centre and radius *OA*.
6. Step off arc *A-A* into 4 divisions such that $AB = BC = CD = DA = 40$ mm and obtain points *A*, *B*, *C*, *D*, *A*.
7. Join base edges *AB*, *BC*, *CD*, *DA*. Also, join slant edges *OA*, *OB*, *OC*, *OD*, *OA*. This is the required development of the pyramid.

PROBLEM 13.10 (FIG. 13.10) *Draw the development of lateral surface of a hexagonal pyramid with a 30 mm base side and a 60 mm long axis, which is resting on its base in the H.P. such that an edge of the base is perpendicular to V.P., when (a) a horizontal section plane cuts the pyramid at a height of 25 mm from the base, and (b) an auxiliary inclined plane whose V.T. makes an angle 60° with H.P., bisects the axis.*

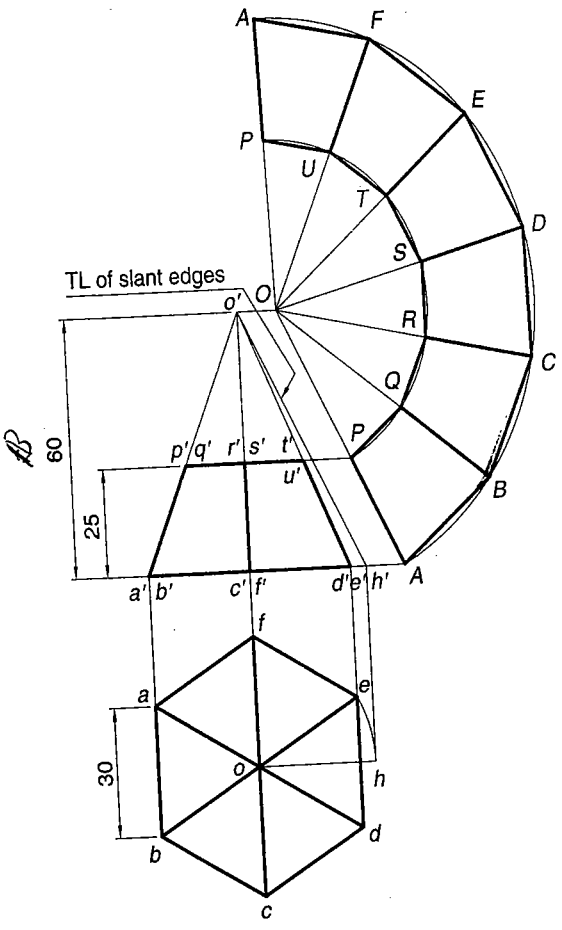


Fig. 13.10(a)

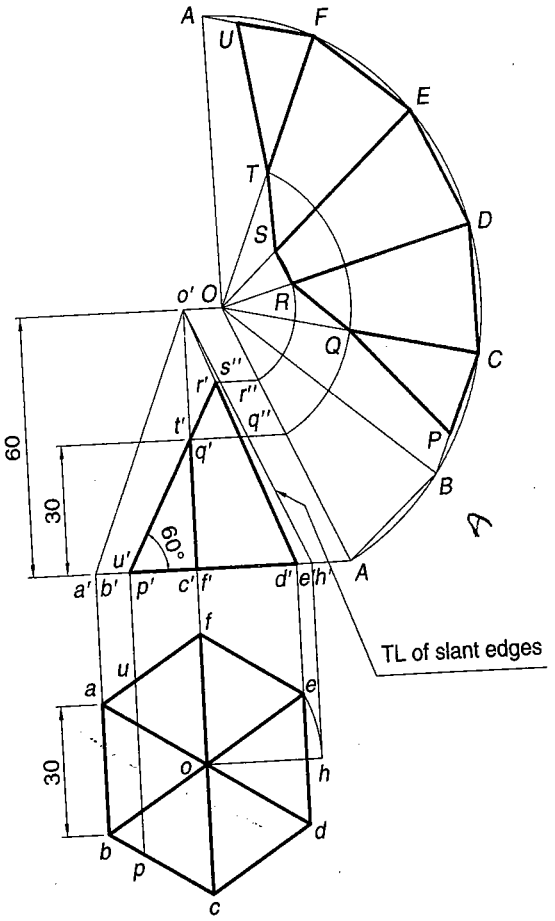


Fig. 13.10(b)

CONSTRUCTION Figure 13.31

1. Assume the pyramid to be uncut, and draw its top view, front view and the development.
2. With centre o and radius oc, draw an arc to meet the horizontal line through o at point h. Project h to obtain h'. Join $o'h'$ which is the required true length.
3. Draw the cutting plane $p'r'$ in the front view inclined at 45° to xy and passing through a point on the axis, 30 mm above the base.
4. Draw the horizontal lines from p', q', r', s' upto OA, and thereafter, rotate them to obtain P, Q, R, S on their corresponding generators. Join these points as shown and obtain the required development.

PROBLEM 13.32 (FIG. 13.32) *A pentagonal pyramid, having base with a 30 mm side and a 60 mm axis, rests on its base in the H.P. It is cut by two section planes which meet at a height of 20 mm from the base. One of the section planes is horizontal, while the other is an auxiliary inclined plane whose V.T. makes 45° with H.P. Draw the development of the lateral surface of the solid when apex is removed.*

CONSTRUCTION Figure 13.32

1. Assuming the pyramid to be uncut, draw its top view, front view and the development.
2. With centre o and radius oc, draw an arc to meet the horizontal line through o at point h. Project h to obtain h'. Join $o'h'$ which is the required true length.
3. Draw the cutting plane $p'q'r'$ in the front view.
4. Draw the horizontal lines from p', q', r', s', t', u' upto OA, and thereafter, rotate them to obtain P, Q, R, S, T, U on their corresponding generators. Note that the point T is the mid-point of chord UM. Join the points and obtain the required development.

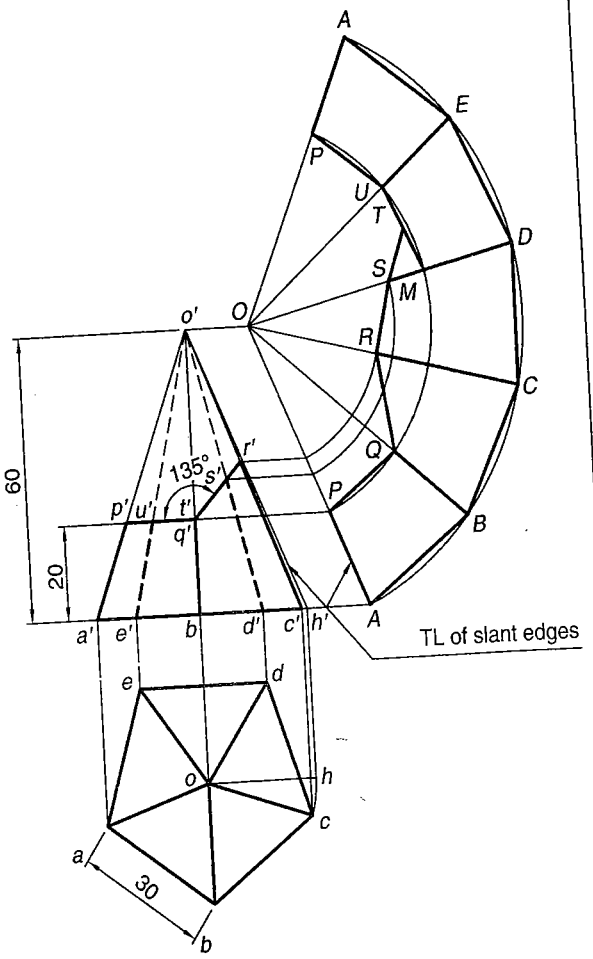


Fig. 13.32

PROBLEM 13.33 (FIG. 13.33) *Figure 13.33(a) shows the front view of a truncated square pyramid having a base with a 40 mm side and a 65 mm long axis, is resting on its base on the H.P. with all the sides of the base equally inclined to V.P. Draw the development of the lateral surface of the retained solid.*

CONSTRUCTION Figure 13.33(b)

1. Assuming the pyramid to be uncut, draw its top view, front view and the development.
2. Draw the cutting plane in the front view, as shown, and name the intersecting points.

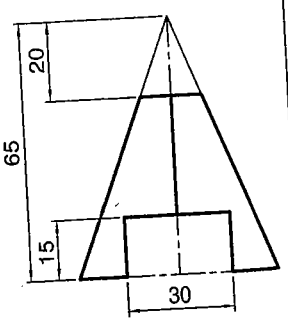


Fig. 13.33(a)